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Theoretical Model for Temperature Distribution Resulting from CW-Laser Radiation Heats up Tumor Tissues

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Abstract

Continuous wave laser radiation is used to heat up tumor tissue where the cells in this tissue are more sensitive to heating. Temperature distribution is connected to hyper thermal therapy. First, the nonlinear bio-heat transfer equation of Penne's type in three dimensions is solved using the approximate analytical Adomian Decomposition Method (ADM), where the thermal conductivity of tissue and blood perfusion are temperature dependent. Next, this model is applied to study the effect of some parameters (laser power, irradiated time) and their impact on laser heat distribution within the tumor tissue. It turns out that, the temperature of the tissue increases with power of laser and with irradiation time. Finally, some important effects in the simulation on laser thermal therapy are discussed.

Keywords: Bio-heat equation, Laser thermal therapy, Tumor tissue, Blood perfusion, Adomian Decomposition Method (ADM).

Introduction

Laser systems have become an important medical equipment that can be possibly in medical surgery to treat or remove tissues as well as to diagnosis cancer. Various types of lasers have used in these processes in the medical industry such CO_2 lasers, diode lasers, dye lasers, excimer lasers, fiber lasers, gas lasers etc., [1].

The continuous wave (CW) lasers are limited in use because the collateral damage on the living tissues needs to be controlled or eliminated. This has prompted the using of pulse wave lasers in medical imaging and therapy applications. Pulse lasers offer the advantage of target delivery of heat energy, i.e. minimizing the spread of heat to the surrounding healthy tissues [2]. Thermal therapy treatment for cancer is a kind of treatment in which tissue is exposed to high temperatures to damage cancer cells with minimal injury to the surrounding tissues, making it much safer than classical treatment therapies [3]. Laser beam interaction with tissues categorized into several mechanisms which include photo-mechanical, photo-thermal and photo-chemical [4, 5] where the photo-thermal effect is considered the most observed one among them. In fact, the majority of the tumor therapy through radiation techniques use either CW laser beam or long pulsed laser. The CW laser beam will produce more residual thermal damage to the surrounding the ablation more than short pulsed CO_2 laser but less depth of ablation, but many researchers found the ability to produce highly localized heating at the desired location which has made the pulse laser more efficient for tumor irradiation than (CW) lasers [6,7].

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In this work, the axial and radial temperature distribution resulted from CW-Laser radiation heats up tumor tissues was modeled theoretically. The obtained results are compared with recent practical and theoretical studies [8, 9]. Well-established investigations have shown that the tissues are sensitive to heating and the temperature required to kill the tumor is depended on the irradiation time and laser power [1, 4]. CO_2 laser is the most widely used in the field of dermatology owing to its suitable wavelength in the mid-infrared (at 10,600 nm) and absorption coefficient in water (around 500 m^{-1}). Therefore, all incident beam energy is well absorbing in tissue water and prevents deeper tissue damage which makes the CO_2 laser as the safest system [10]. Many experiments have found out that temperature elevation to at least 56 C^{o} for one second or more should be sufficient to kill cancer cells [11, 8]. Thus, there is no consensus in the literature on the exact extent of the temperature rise and the exposure time necessary for complete tumor ablation. Simply in present model, it is assumed that, the beam applies heat at point in the center of tumor only. In this research, because the tissue blackbody intensity is much smaller than the incident laser intensity the radiation emission from the tumor tissue was neglected [2].

Mathematical Model

In this model, we utilize the non-liner Penne's bio-heat equation in three dimensions i.e. the general heat diffusion equation with additional terms for perfusion of blood, thermal conductivity, as temperature-dependent functions [12, 13] as follows:

$$\rho_t c_t \frac{\partial T(r,t)}{\partial t} = \nabla\kappa(T)\nabla T(r,t) - \omega_b(T)\rho_b c_b(T - T_a) + Q_m + Q(r,t) \quad (1)$$

where ρ_t and c_t are the density [kg. m^{-3}] and the specific heat of the tissue, respectively; ρ_b and c_b denotes the density and specific heat of the blood; r contains the Cartesian coordinates x, y, and z. $\kappa(T)$ is the temperature-dependent thermal conductivity; and $\omega_b(T)$ is the blood perfusion. The value of blood perfusion represents the blood flow rate per unit tissue volume. T_a is the blood temperature in the arteries supplying the tissue and it is often treated a constant at 37 C^{o}. $T(r,t)$ is the tissue temperature Q_m is the metabolic heat generation, and $Q(r,t)$ is the distributed volumetric heat source due to externally applied spatial heating, in this literature laser heat flux is considered as a heat source imposed on the top side of the slab (see figure 1), with,

$$Q(r,t) = \frac{2\alpha P_o e^{-\alpha z}}{\pi\omega_o^2} e^{-2\frac{(x^2+y^2)}{\omega_o^2}} \quad (2)$$

Where α is the tissue absorption coefficient, P_o is the laser power intensity in watts at the surface, ω_o, is the waist radius [14, 15].

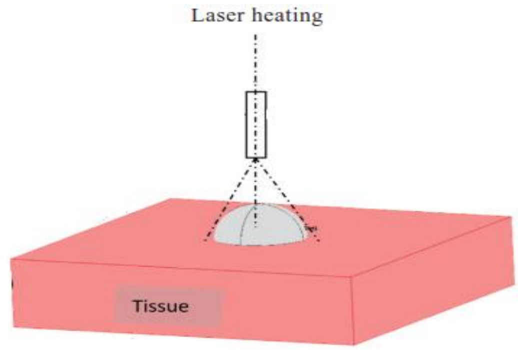

Figure 1. A schematic of a laser-irradiated tumor tissue

The left-hand side of Equation (1) represented the rate of thermal energy which absorbed per volume unit, while the biological tissue has included. According to Fourier's law, the first term in the right-hand side is the rate of heat conduction. In addition, the second term represents the rate of heat convection through blood vessels. The convective heat exchanges by blood circulation insures thermal regulation throughout the body. Blood enters the tissue at the arterial temperature T_a, exchanges a certain amount of energy which is equivalent to $\omega_b(T)\rho_b c_b(T - T_a)$ bring the blood temperature to the level of that of the tissue. In this work, $\kappa(T)$ the thermal conductivity and $\omega_b(T)$ blood perfusion rate is assumed to be linear function of temperature [12, 16-18]:

$$\kappa(T) = \kappa_a + \beta(T - T_a) \tag{3}$$

$$\kappa_a = 639\ W/mK\,, \quad \beta = -0.000758/K\,, \quad T_a = 300\ K$$

$$\omega_b(T) = a_1 + a_2 T \tag{4}$$

Where $a_1 = 0.0005$ and $a_2 = 0.0001$

The distribution of heat flow in a three-dimensional space is governed by the following initial boundary value problem;

$$0 \le x \le a\,, 0 \le y \le b\,, 0 \le z \le L$$

Boundary conditions,
$$\left.\begin{aligned} T(0,y,z,t) = u(a,y,z,t) = T_o \\ T(x,0,z,t) = u(x,b,z,t) = T_o \\ T(x,y,0,t) = u(x,y,L,t) = T_o \end{aligned}\right\} \tag{5}$$

And, initial conditions, $$T(\vec{r},0) = T_o \tag{6}$$

Where, $T \equiv T(\vec{r},t)$ is the temperature of any point located at the position $\vec{r}$ of a rectangular volume at any time t.

Solution of equation (1) by (ADM)

Adomian Decomposition Method [16,19], known as semi-analytical (or approximate analytical), have recently been proposed for solving nonlinear problems. This method was first developed by Adomian [20] and has been used by many researchers [21]. Several applications of this method has been reported and one of the most powerful of these tools has been employed numerously to solve linear and nonlinear equations in physics [22].

The formulation of (ADM) [19] for solving equation (1) is as follows:
Dividing equation (1) by $\rho_t c_t$, one can get;

$$\frac{\partial T(r,t)}{\partial t} = \frac{1}{\rho_t c_t} \nabla \kappa(T) \nabla T(r,t) - \frac{\omega_b(T)\rho_b c_b(T-T_a)}{\rho_t c_t} + \frac{Q_m+Q}{\rho_t c_t} \quad (7)$$

By rewriting equation (7) in an operator form by;

$$L(T) = N(T)$$

Where, the differential operator L is given by:

$$L(.) = \frac{d}{dt}(.)$$

And;

$$N(T) = \frac{1}{\rho_t c_t}\left[\frac{\partial}{\partial x}\left(\kappa(T)\frac{\partial T}{\partial x}\right) + S(T) + Q(r,t)\right] \quad (8\text{-a})$$

$$M(T) = \frac{1}{\rho_t c_t}\left[\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right)\right] \quad (8\text{-b})$$

$$F(T) = \frac{1}{\rho_t c_t}\left[\frac{\partial}{\partial z}\left(\kappa(T)\frac{\partial T}{\partial z}\right)\right] \quad (8\text{-c})$$

With;

$$S(T) = \omega_b(T)\rho_b c_b(T - T_a) \quad (9)$$

The inverse operator L^{-1} is considered as integral operator defined by;

$$L^{-1}(.) = \int_0^t (.)dt \quad (10)$$

By operating L^{-1} in the righthand side of equations (8-a, b, c) and using of the initial conditions,

T(x ,y, z, 0)= 37 C° , leads to ;

$$T(x,y,z,t) = T(x,y,z,0) + L^{-1}[N(T) + M(T) + F(T)] \quad (11)$$

The solution $T(x,y,z,t)$ introduced by ADM in an infinite series form as;

$$T(x,y,z,t) = \sum_0^{\infty} T_n(x,y,z,t) \quad (12)$$

where, the components $T_n(x,y,z,t)$ are determined recurrently, the nonlinear operators $N(T), M(T), F(T)$ can be decomposed into an infinite series of polynomials which are given by ;

$$N(T) = \sum_{n=0}^{\infty} A_n \; ; \quad M(T) = \sum_{n=0}^{\infty} B_n \; ; \quad F(T) = \sum_{n=0}^{\infty} C_n$$

where A_n, B_n, C_n are the Adomian polynomials which has been introduced by the Adomian himself by the formula.

$$A_n(T_0, T_1, T_2, \dots, T_{n-1}) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i T_i\right)\right]_{\lambda=0} \qquad n = 0,1,2,\dots \quad (13\text{-a})$$

$$B_n(T_0, T_1, T_2, \dots, T_{n-1}) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[M\left(\sum_{i=0}^{n}\lambda^i T_i\right)\right]_{\lambda=0} \quad (13\text{-b})$$

$$C_n(T_0, T_1, T_2, \dots, T_{n-1}) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i T_i\right)\right]_{\lambda=0} \quad (13\text{-c})$$

If we now using the above equations (8-13),

$$T(x, y, z, t) = T_0(x, y, z, 0) + L^{-1}\left[\sum_{i=0}^{\infty}(A_n + B_n + C_n)\right] \quad (14)$$

According to Adomian, $T_0(x, y, z, 0)$ is identified with the following recurrence,

$$T_{n+1}(x, y, z, t) = L^{-1}(A_n + B_n + C_n), \quad n \geq 0 \quad (15)$$

The solution will be approximated by a series of the form;

$$\Phi_N(x, y, z, t) = \sum_0^N T_n(x, y, z, t) \quad (16)$$

So that a series solution for equation (1) is obtained.

By assuming the thermal conductivity of tissue and blood perfusion are temperature dependent, the generated solution is in the general form and it is more realistic as compared with the method of simplifying the physical problems [12] because the method does not resort to linearization or assumption of weak nonlinearity [19].

Result and discussion

When tumor tissue is subjected to CO_2 laser beam at specific wave length, the light penetrates into a certain distance (nearly 15 – 20 μm) [23,24]. The absorption power from the tissue will be changed to heat transporting through skin by heat conduction. Cell proteins began to dissolve and also the RNA, DNA and all the cell contents at a temperature of 40 C^o so the laser reactions with the tissue can cause tissue necrosis and blood coagulation and change in the structure. The change degree depends on laser power density, surface area subjected to the radiation, the wave length, speed of laser diffusion through the tissue, so thermotherapy can affect tissue temperature and its regulation [25, 26, 27].

In the present work, temperature distributions in a tissue medium during a (CW) laser irradiation has been obtained semi-analytically by (ADM)). Radial and axial temperature distribution in three dimensions obtained in tumor tissue samples for the case of CO_2 laser beam with wavelength of 10.6 μm to treat a skin tissue of $2 \times 2\ mm^2$ with a thickness of $4\ mm$ and there is swelling on the skin surface of diameter 1mm. The laser beam subjected to the tumor circular position when regarded that beam contrition approximates to the surface area of the tumor subjected to ($\approx 12\ mm^2$). In particular, $z(m)$ represents depth into the tumor tissue, so that $z = 0$ at the surface and $z > 0$ depth inside the tumor tissue. Further, $r(m)$ represents the transverse distance which is the distance from the beam axis. The tumor irradiation technique is used continuous wave (CW) or long pulsed laser which has recently become preferable for this application. The ability to produce high localized heating at the desired location has made (CW) lasers attractive in heating tumor tissue using suitable energy input and irradiation time. The constants of laser source and tumor tissue in the Tables (1) and (2) were used in this model [8].

Table 1. Specifications of laser source CO_2 [8]

Parameters	Value
Wavelength	$10.6\,\mu m$
Divergence	2 $mrad$
Output beam diameter	6 mm
Output mode	$TEMoo$
Output power level in (CW) mode	(1-50) $watt$
Max. output power	70 $Watt\ Max$

Table 2. Constants and symbols used in this study [9]

Description	Symbol	Value
Density of blood	ρ_b	1000 Kg/m^3
Specific heat of blood	C_b	4200 $J/Kg\,.K$
Arterial blood temperature	T_b	37 C^o
Thermal conductivity of tumor	K_t	0.5 $W/m\,.K$
Density of tumor	ρ_t	1050 kg/m^3
Specific heat of tumor	C_t	3600 $J/kg\,.K$
Blood perfusion rate of tumor	ω_b	$6x10^{-3}/s$
Metabolic heat source	Q_m	33800 W/m^3
External heat source	Q	W/m^3(equation 11)
Body core temperature	To	37 C^o
Absorption coefficient	A	500 cm^{-1}
Power	P_o	W
Spot size	$\omega(z)$	mm
Spot area		mm^2

The following problems are studied via the solution of equation (1):

1. Surface temperature distribution in three dimensions has located in the middle of the treated area using the estimated program in this study. However, this area were explained previously in Figure (2).
2. The relation between the heat distribution in tumor tissue and the penetration depth of the laser irradiation, fixing the irradiation time, beam size and change laser power. Notice that the temperature increases when the laser power increases. In addition, highest temperature is at the tumor surface as in Figure (3). These results agree with the others [23, 8].

3. The relation between heat distribution in tumor tissue and laser exposure time fixing laser power, beam size and depth of the tumor. Notice that, the highest temperature is at the longest time of exposure and it decreases gradually as depth of the tumor increases as seen in Figure (4). This result agrees with those obtained by other researchers [8, 9].
4. The relation between heat distribution and depth in the tumor tissue given in three dimensions when laser power and exposure time are fixed while spot size of laser beam is shown in Figure (5). It is clear that the temperature of tumor is inversely proportion with spot size of the laser beam. This result agrees with those obtained by other researchers [8].
5. Filled temperature contours for the case of laser beam focus on the surface of tumor tissue; represent the relation between spot size of laser beam and the temperature. It is noticed from Figure (6) that the temperature is inversely proportioned with the spot size of the laser beam. It is at the highest level when the spot size is at the lowest level.

The obtained results indicate that the ideal use of laser system must be done after understanding the relation between the laser beam power and spot size and the biology of the tumor tissue. On the other hand, the understanding of the effective physical factors in the laser power which have great heat role, i.e., controlling with unwanted effects.

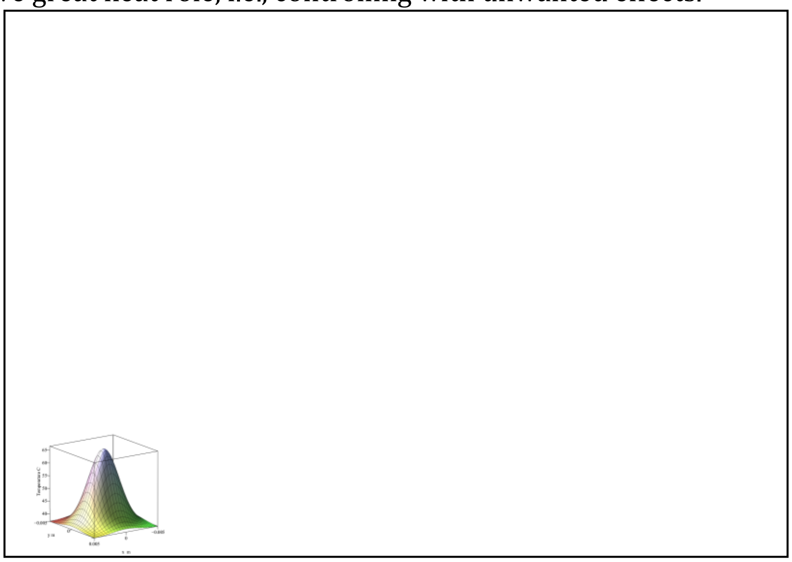

Figure 2. Surface temperature distribution in three dimensions taken from the middle of the work piece obtained from the computer program at irradiation time t=1sec, laser power 10 W. It is clear from the figure that the higher temperature is at the center of the tumor.

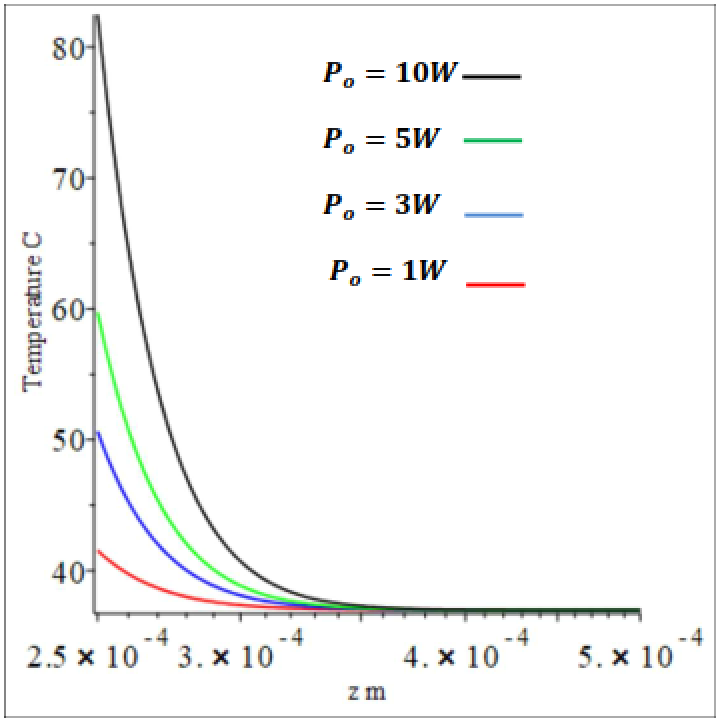

Figure 3. Radial temperature distribution with depth into the tumor tissue z for different laser power P_o at the center of the laser beam band (r = 0) and irradiation time t = 0.5 sec .

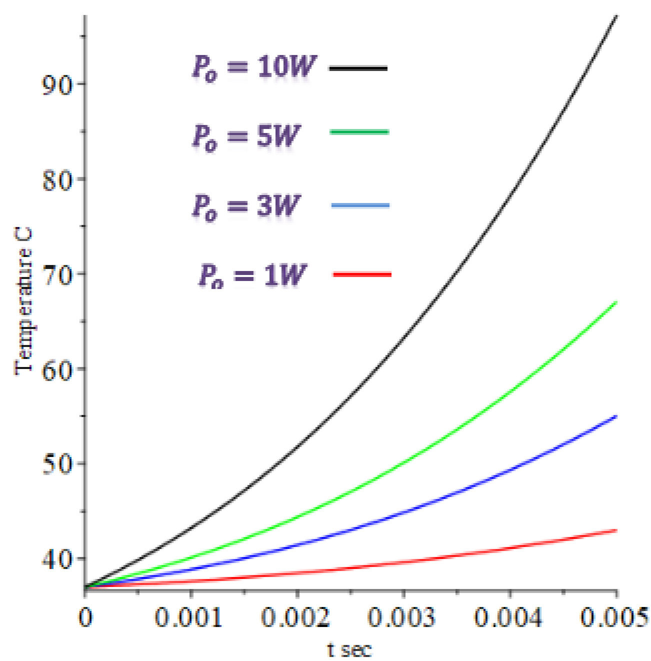

Figure 4. Radial temperature distribution against irradiation time t for beam focusing at the surface of tumor tissue $z = 0$ with different laser power P_o and at the center of the laser beam band (r = 0).

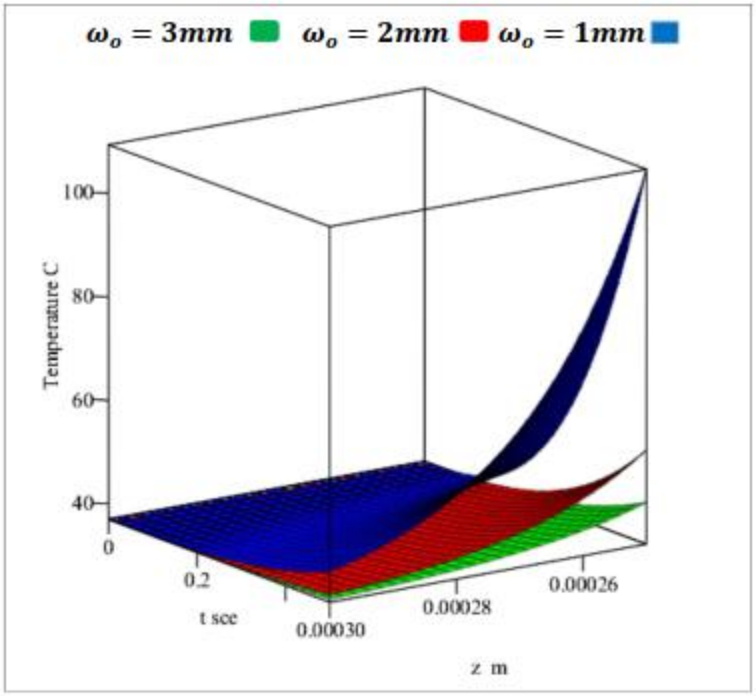

Figure 5. The relation between temperature against irradiation time and penetration depth in to the tumor tissue when the laser power is (1 W) for different spot size of the laser beam.

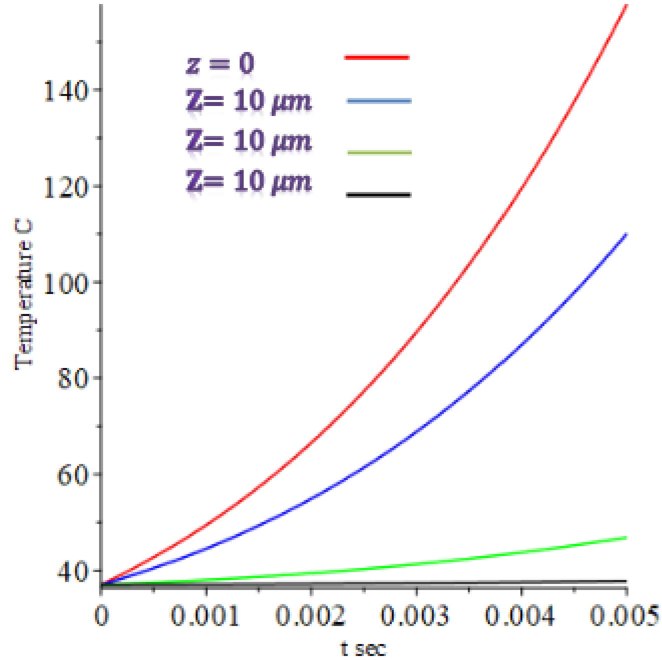

Figure 6. Radial temperature response against irradiation time for different penetration depths in to the tumor tissue for spot size = 2mm and laser power 5 W

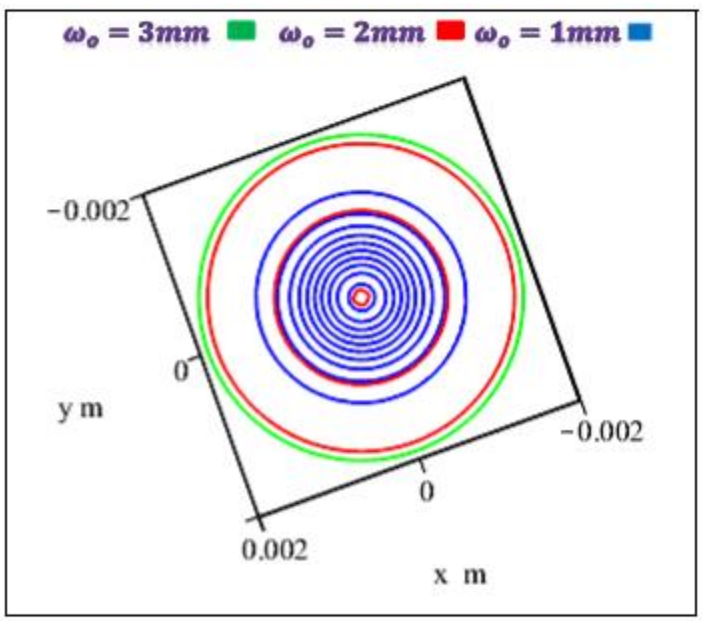

Figure 7. Filled temperature contours, represent the relation between spot size of laser beam and temperature, for the case of laser beam focus on the surface of tumor tissue.

Conclusions:

Result of this study is concentrated on the physical features that related to the reaction of the (CW) beam of CO_2 laser with a tumor tissue and to know the biological effects on it. It can summarize; the most important conclusions from this theoretical study after comparing them with previous practical and theoretical results as shown down.

In case of using high power (5–10) W, after fixed the exposure time, the depth inside the tissue has increased. However, the opposite results were indicated in the low power status (1-3) W. While, using different exposure time with constant power density of laser, the tissue damages will increase as the exposure time increases which may lead to research for certain laser power density to each case wanted to be treated, that is very important to decrease the un-desired side effect much as possible. Herewith, the high-power density of CO_2 laser in case of using it in thermal therapy according to direct heating and great damage in the tissue.

Moreover, using ADM process by building the required program make the process more actual when there are less summations and no need to take long measuring and reduce the time wasting. However, this will encourage the authors to study the pulses laser therapy in the same mathematical model in the future.

In the next study, tumor can be indirect heating with low power density of radiation using gold nano-particles based on Plasmon Resonance Phenomenon (PRP). This to increase possibility by solve the space and the time dependent bio-heat equation with different conditions using the same mathematical treatment.

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