Description of yrast states and deformation changes in tantalum nuclei (Ta)

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Abstract

The gamma – ray studies of 167Ta nucleus will be studied and several rotational decay sequences are identified. In these nuclei values for the effective moment of the inertia are suggested which reproduces the energies of the non–yrast states, that are populated in decay of the isomer. The results are discussed in comparison of the energy level patterns for the tantalum isotopes with exiting data in the neighboring nuclei and thus has been done by studding the changes in deformation through the so called "Gauge Plots".

Keywords: Nuclear structure, Yarst lime, Deformation, Gauge Plots.

1. Introduction

Transitional nuclei with proton numbers Z= 64-68 and neutron numbers N = 82-86 are presumably spherical at low spins and deformed at high spins, an oblate deformation was inferred from the slops of the yarst line. according to Bohr and Mottelson (1) the yarst line of a system of single particles aligned along the axis of symmetry depends linearly on I (I+1), The slops being given by \(\hbar^2/2\zeta\). Values of \(\zeta\) in excess of the rigid-rotor value, deduced from the slope of high-spin sections of the yarst line, are then taken as evidence for a contribution to \(\zeta\) from the oblate-shaped core. in a pioneering study Del Zoppo et al. (2), demonstrated the existence of high spin isomers that decay by \(\gamma\) – cascades of high multiplicity in several of these nuclei. The transitional N=88 isotopes are found to characterized by a small prolate deformation at low spin , but a switch to oblate aligned particle non-collective yarst states occurs around I = 30 \(\hbar\), which is followed by return to moderate collectivity at yet higher spins (I > 40 \(\hbar\)).

The studied nucleus 167Ta with the valence neutron outside the N = 82 shell is essentially spherical in its ground states and its low spin structure is understood on basis of the spherical shell model (3). The 167Ta nucleus was studied (4, 5, 6 and 7) through measurement of the gamma radiation from the 142Nd (30Si, pxn) reaction.

The light Ta isotopes lie near the transitional region between deformed and spherical shapes. They are characterized by small quadruple deformation and rather shallow minima of the potential energy surface with respect to the \(\gamma\)-deformation (2). It was our aim to study the dependence of band crossing frequencies aligned angular momentum and signature splitting on the occupation of the different proton states.

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2. Theoretical Calculation and Discussion

2.1. Shape of the yrast line

The experimental yrast level energies versus \( I(I+1) \) for Ta isotopes are given in Fig. (1).

![Diagram showing yrast line for Ta isotopes]

Fig. (1): Excitation energies for yrast states with the corresponding \( I(I+1) \).
The excitation energies are reproduced on an average by straight line which start at \( I = 0 \) with an excitation energy \( E_0 \) until \( I = (41 / 2) \hbar \). The fluctuations indicate that the yrast angular momentum dose not originate from collective rotation where one would expect a smooth behavior of the yrast line.

![Number of states](image)

**Fig. 2:** Number of calculated states within the first 1 MeV above the yrast line shown as a function of the angular momentum.

A part of the fluctuations seen in Fig. (2) are due to accidental variation of the level density for a single deformation. For example, the drops of the level density of \( I = 20 \hbar \) can be traced back to a particularly low energy of the yrast configurations with these spin values. The \( I (I+1) \) dependence is characteristic for the rotation of a rigid body or the average yrast line of the Fermi gas (1). The good agreement for high spin values may be taken as evidence of the fact that Fermi gas model is a good approach for the few particle excitation in this nucleus \(^{167}\text{Ta}\). A part of the fluctuations seen in this figure are due to accidental variations of the level density for a single deformation.

In Fig. (1) the following interesting features can be observed: With decreasing mass number (decreasing deformation and decreasing moments of inertia see Fig.(1), the backbending become sharper, which reflects a decreasing interaction energy. In \(^{167}\text{Ta}\) the plot show strong oscillations of moment of inertia \( (2\zeta/2\hbar) \), which can be traced to the admixture in the ground band, its decoupling parameters. Also this strong staggering shows the \(^{167}\text{Ta}\) may be soft rotor against \( \gamma \)-deformation. Since the clustering of high-J orbital around the Fermi levels tends to decrease the slop of the yrast line for spins less than the maximum possible with valence nucleons. For larger spins, on the other hand the necessity of promoting particles across \( Z = 64 \) or \( N = 82 \) gap will result in a steeper yrast line. An example of the latter effect may be seen in the sharp increase in slop of the yrast line fig. (1), just beyond spin \( 43/2 \hbar \). This change in the slope of the yrast line at high spins was can be interpreted as a shape transition from spherical to oblate. The difference between \(^{167}\text{Ta}\) level energies and the rotating liquid-drop energies \( \text{ERLD} =3.22+0.00781(I+1) \text{ MeV} \) [9] is plotted in Fig. (3) [See also table 1].
Fig. 3: Difference between level energies $E_x$ and a rotating liquid drop energies are plotted as a function of spin.

<table>
<thead>
<tr>
<th>I</th>
<th>$^{167}\text{Ta}$</th>
<th>$^{169}\text{Ta}$</th>
<th>$^{171}\text{Ta}$</th>
<th>$^{173}\text{Ta}$</th>
<th>$^{175}\text{Ta}$</th>
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<tbody>
<tr>
<td>2</td>
<td>-1.1964</td>
<td>-0.1621</td>
<td>-0.3092</td>
<td>-0.2361</td>
<td>-0.0491</td>
</tr>
<tr>
<td>4</td>
<td>-</td>
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<td>-0.5482</td>
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<td>-0.0981</td>
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<tr>
<td>6</td>
<td>-</td>
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<td>-0.5241</td>
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<tr>
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<td>-0.1551</td>
<td>-0.8222</td>
<td>-0.6171</td>
<td>-0.1961</td>
</tr>
<tr>
<td>10</td>
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<td>-0.4870</td>
<td>-1.0030</td>
<td>-0.6771</td>
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</tr>
<tr>
<td>12</td>
<td>-0.1616</td>
<td>-0.4321</td>
<td>1.2360</td>
<td>-0.7141</td>
<td>-0.2770</td>
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</table>

Table 1: Results of the Fermi energy ($\lambda$) with the number of neutrons for isotopes $^{167}\text{Ta}$ [13], $^{169}\text{Ta}$ [14], $^{171}\text{Ta}$ [15], $^{173}\text{Ta}$ [16], $^{175}\text{Ta}$ [17].
Two main features emerge from the direct inspection of the figure: (1) up to the highest states, the yrast line remains of single-particle character and the calculation reproduce the main trends very well, and (2) there is no hint in the data of any new collective structure analogous to the one seen at high spin in 167,168W. This feature is most apparent in this figure where there is no sign of a sharp downward trend of the data points in 167Ta as was the case for 167,168W. This finding is consistent with the fact that no fast transitions were observed in 167Ta. Thus, it appears that the 167Ta collective structures, if present at all, are located at higher excitation energy and/or spin than in 167Ta it seem that small oblate deformation ($\beta = -0.1$ to $-0.2$); shell effects and pairing correlations are responsible for the excess of observed moments of inertia in 167Ta over the rigid sphere values. It can be calculated $\zeta$ effect from Mottelson and Valatin equation $2\zeta / h^2 = 36 /A^{3/4}$ [10], assuming the fact that the pairing vanishes and the moment of inertia reaches the rigid sphere values ($h^2/2\zeta = 114$ MeV$^{-1}$) at a deformation appropriate to $\omega = 0$.

2.2. Gauge plots and deformation changes

The changes in deformations can be explored through the so-called "gauge plots" [11] of neutron number ($N$) versus Fermi energy ($\lambda$), in analogy to the plots of $I$ versus $\omega$, which are frequently used for investigating rotational bands in ordinary space. In such a plot one examines the difference in excitation energy $E(I)$ for the ground-band members with spin ($I$) in pair of neighboring even-even nuclei. This energy difference, together with the two nucleons separation energy $S_{2n}$, specifies the Fermi energy ($\lambda$) as a function of spin. The Fermi energies are calculated from the relation:

$$\lambda(N, I) = 1/2 \left[ E(N+1, I) - E(N-1, I) - S_{2n} \right]$$

Where ($N$) is the (odd) neutron number between the two even isotopes, which are compared. The separation energy ($S_{2n}$) for each neutron number is taken from Ref. [12]. Figure (4) gives the results for the Ta nuclei calculated from the experimental energies [13, 14, 15, 16, and 17] of the even isotopes with 94 to 106 neutrons [see also table 2]. In this figure we see a pronounced irregularity around neutron number 99, which reminds very much about the back bending or up bending (depending on $I$) behavior. The slope of the curves $N(\lambda)$ reflects the level density at the Fermi surface. An upbend (for example) in this curve (i.e. an increase of slop in a small energy region) indicates a sudden increase in level density. This tends to be energetically unfavorable and must therefore be compensated, most likely by a change in deformation, typically an increase. Thus, in such a case, the upper branch in the $N(\lambda)$ curve is called the deformed branch (e.g. = 99) in fig. (4).
Fig. 4: Backbending plot showing the neutron number vs. the Fermi energy for each spin in Gauge space for even Ta isotopes.

<table>
<thead>
<tr>
<th>I</th>
<th>(E_x - E_{RLD})</th>
<th>(E_{RLD})</th>
<th>I</th>
<th>(E_x - E_{RLD})</th>
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<tr>
<td>2</td>
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<td>3.34</td>
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<tr>
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<td>11.5</td>
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</tr>
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<td>0.58</td>
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<td>8.5576</td>
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</table>

Table 2: The value of the difference between the experimental energy levels and rotating liquid-drop model energy. \(E_{RLD} = 3.22 + 0.007811(I+1)\) MeV [Ref.1] corresponding to the angular momentum.
3. Conclusions

The calculation provides two values for the effective moment of inertia, which reproduces the energies of the non-yrast states that are populated in the decay of the isomer. In $^{167}$Ta a change in the slope of the yrast line at high spins was found and interpreted as a shape transition from spherical to oblate form. Information on the energy level patterns through the so-called gauge plots for the Ta isotopes with neutron numbers 94-102 clearly shows the backbend between the spherical and the deformed nuclei.

References