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Comparative Study of Transverse Vibration and Mechanical Properties of Aluminium, Al 7020 Alloy, and MWCNTs Reinforced Aluminium Nanocomposites

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ABSTRACT

Alloys that mostly composed of aluminium are widely used in structural engineering especially in the aircraft industry. However, cyclic vibration may generate microcracks which lead to failure. Mechanical vibration is one of the most popular issues in any working machine. Besides, the vibration energy may transfer to the other portion of the structure as any mechanical waves causing noises, loose parts, heat, and wear. Structure damping and vibration isolation are the main two solutions to solve this problem. Metals behave like viscoelastic materials that made them a candidate to serve as dampers. Therefore, this paper investigates the transverse vibration and mechanical properties of aluminium, Al7020 alloy (Al-Zn-Mg alloy) and aluminum-multi-walled carbon nanotubes (Al-MWCNTs) nanocomposites. The samples were prepared using an open mould casting approach with flex. Experimental comparative study of dynamic behaviour and mechanical characteristics for the prepared samples were investigated. Transverse vibration test at different frequencies (motor rotating speed 0-3000 rpm) with and without loading, tensile test, flexural bending, and Vickers hardness tests were determined. Surface morphology and chemical analysis of as a cast prepared samples were characterized utilizing scanning electron microscope (SEM) with energy dispersive spectroscopy (EDS) and optical microscope. The obtained results revealed different dynamic behaviour and mechanical properties of prepared samples due to the different microstructure effect generating from the addition of materials (alloying element and reinforcing materials). Al7020 alloy showed the highest ultimate tensile strength, axial stiffness, flexural strength, bending modulus, fracture toughness, and hardness compared with the other samples. Al-MWCNTs nanocomposites revealed minimum ultimate tensile strength, axial stiffness, and elongation % at the break, bending modulus, fracture toughness and maximum deflection. Moreover, the dynamic behaviour of all samples is dissimilar under transverse vibration test when applied load and frequency were changed. The Al-MWCNTs nanocomposites exhibited the most stable structure under transverse vibration test at maximum applied load and higher frequencies.

Keywords: Transverse Vibration, Al 7020 alloy, Aluminum, MWCNTs, Nanocomposites.

1. INTRODUCTION

Nanotechnology is a newly discovered field, which describes and manipulates the unique properties of materials in nanoscale, to enhance their capabilities with science, engineering, and medicine applications [1,2]. The nanocomposite is one of the most widespread applications of nanotechnology. It is a solid material involving at least one phase or more has one, two or

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dimensions less than 100 nm, or consist of structures having distances between the different phases in nano-scale [3].

Numerous researches and versatile applications dealt with the nanocomposites utilizing metal, polymer, and ceramics as a matrix reinforced with different structures of nanomaterials. Organic and nonorganic nanoparticles can be used as reinforcing filler. Due to the interest of current work on aluminium in the damping system, many studies related to aluminium as a pure, alloy and nanocomposites have been taken into account when executed this work.

Mechanical vibrations are some of the issues which usually happen when machines operate. The vibration sound transfers to the structure causing noises, loose parts, wear, and other problems. High-frequency vibration or long-term weak frequency vibration can cause machinery failure. Internal vibration damping (structure damping) and vibration isolation are the main two approaches to reduce the vibration [4,5]. Structure damping is self-damping of vibration that means energy dissipates through materials itself as dislocations or other types of defects. It is considered as an important design parameter especially for structures in the aeroplane, oil and automobile industry [6]. The cyclic oscillation has the main role of vibration theory when potential energy transforms into kinetic energy and returns cyclically [7]. Vibration dampers are mainly metals and polymers, due to their viscoelastic character [8]. Metallic materials for damping applications involve smart alloys, ferrous alloys and other [9]. Ferromagnetic alloys include those based on iron (cast iron, steel, Fe-Ni-Mn, Fe-Al). While damping alloys, the damping mechanism attributed to microstructural, which considered the most used damping metals due to low cost like aluminium alloyed with (germanium, cobalt, zinc, copper, silicon, or alloys 6061, 2017, 7022 and 6082), zinc alloys, lead alloys, tin alloys, titanium alloy, nickel superalloys, zirconium alloys, copper and magnesium alloys. In addition, metal-matrix composites can be considered one kind of damping metals like (Al/SiC, Al/graphite, Mg/carbon, and NiAl/AIN). Metal composites provide vibration damping due to the interface between reinforcement and matrix, leading to enhance the damping capacity and stiffness which is responsible for vibration reduction [8]. Aluminium alloy and composites have found considerable applications in aerospace, automotive and transportation industries due to its high mechanical properties compared with pure aluminium [9-13].

Rohatgi *et al.* (1976) enhanced damping capacity and machinability of cast pure aluminium and its alloy by adding graphite particles. Torsion pendulum was utilized to determine damping capacity at constant strain. The results showed that graphite increased the damping capacity; the rises of damping increases with graphite content reaching to cast iron damping behaviour at higher content of graphite [14]. Crawley *et al.* (1987) studied damping behaviour in the aluminium beam and aluminium matrix composites. Al2024, Al6061, graphite/Al6061 and graphite/AZ91C magnesium matrix were tested with different frequencies and stress. It was found that alloys behaviour differs from the predicted results by Zener Modal [15]. Kuzumaki *et al.* (1998) incorporated carbon nanotubes (CNTs) in aluminium by hot-press and hot-extrusion methods. From TEM observations, the CNTs in Al composite are found not damaged during preparation. Also, the ultimate tensile strength of the nanocomposites was slightly affected by the annealing time at 873 K, while pure aluminium was significantly decreased with time [16]. Deng *et al.* (2007) published two papers about 2024Al and nanocomposites. They prepared aluminium reinforced with CNTs using isostatic pressing then hot extrusion. They noticed that CNTs react with Al forming Al_4C_3 phases when heated above 656.3°C. The CNTs adding enhanced mechanical properties of nanocomposites significantly and tensile strength and Young's modulus was better than pure 2024 Al alloy. The results revealed CNTs enhance the high damping capabilities of a metal matrix at an elevated temperature without losses in the mechanical strength of a metal matrix [17,18]. Also, Hussain *et al.* (2017) found that composites' density and hardness improved with the increasing of CNTs amount into pure aluminium matrix [19]. Zhou *et al.* (2017) enhanced load transfer between MWCNTs and Al matrix using interfacial reactions by heat treatment forming aluminium carbide which acts as anchor hinder of slippage [20]. Numerous reviews were reported that focuses on properties and application of metals matrix composites and nanocomposites [21].

Aluminium has been widely utilized in structural engineering and aircraft manufacturing due to lightweight, ductile, and corrosion resistance. However, aluminium is not tough enough during vibration as stainless steel. Therefore numerous diffident alloys and composite were created during the last decades. Many different factors affect the damping of metals and alloys. Some of them are environmental like temperature, frequency, stresses, and fatigue cycles. While intrinsic factors that relate to the material structure itself also have a significant influence on internal dampings like grain size, pore size, and content, miscibility of alloy phases, bonding strength among amount composite components, etc. Therefore in this study, it was utilized the same matrix (aluminium) in three different cases (pure aluminium, 7074 aluminium alloy, and CNT reinforced aluminium) to show the effect of internal material structure on the transverse vibration and mechanical properties.

This work aims to compare the dynamic behaviour and mechanical properties of Al, 7020 Al alloy and Al-MWCNTs composites. Transverse vibration at different load, tensile test, flexural test and hardness are determined. Energy dispersive spectroscopy (EDS), scanning electron microscope (SEM) and optical microscope are also utilized for characterization.

2. EXPERIMENTAL WORK

2.1 Materials

Commercially pure aluminium wire and 7020 Al alloy plates were purchased from the public market. The chemical specification was investigated by EDS as shown in Figure 1. Non-functionalized multi-walled carbon nanotubes were pre-prepared using water-assisted chemical vapour deposition (WACVD) with specification shown in Table 1 and SEM and TEM characterization as shown in Figure 2.

Table 1 Specification of pre-prepared MWCNTs

MWCNT	Description
Production method	CVD
Available form	Black powder
Diameter	Outer diameter: 20-40nm
Length	< 10μm
Purity	>97%
Metal particles	<2%
Amorphous carbon	<1%
Specific Surface Area	212 m^2/g

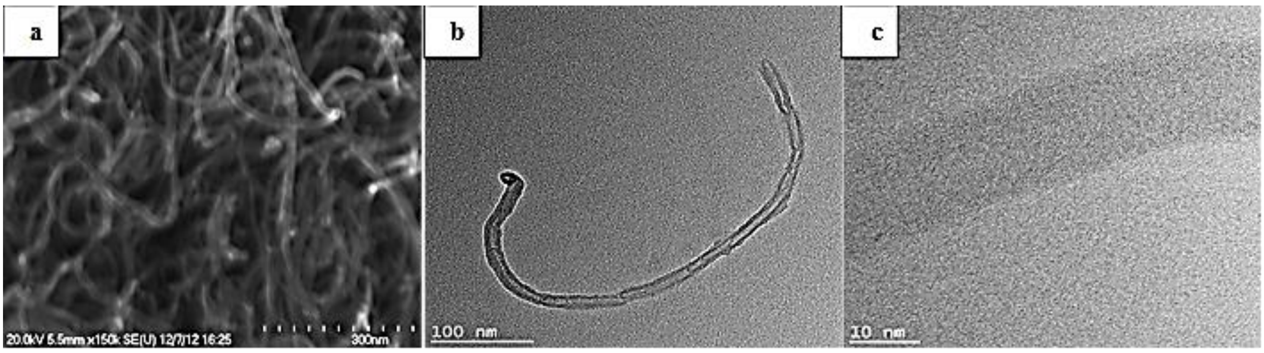

Figure 1. a) SEM and b, c) TEM images of MWCNTs.
and b, c: TEM images of MWCNTs.

Element	Wt.%
O	**2.24**
Mg	**1.28**
Al	**91.19**
Si	**0.29**
Ca	**0.24**
Cr	**0.27**
Cu	**0.42**
Zn	**4.08**

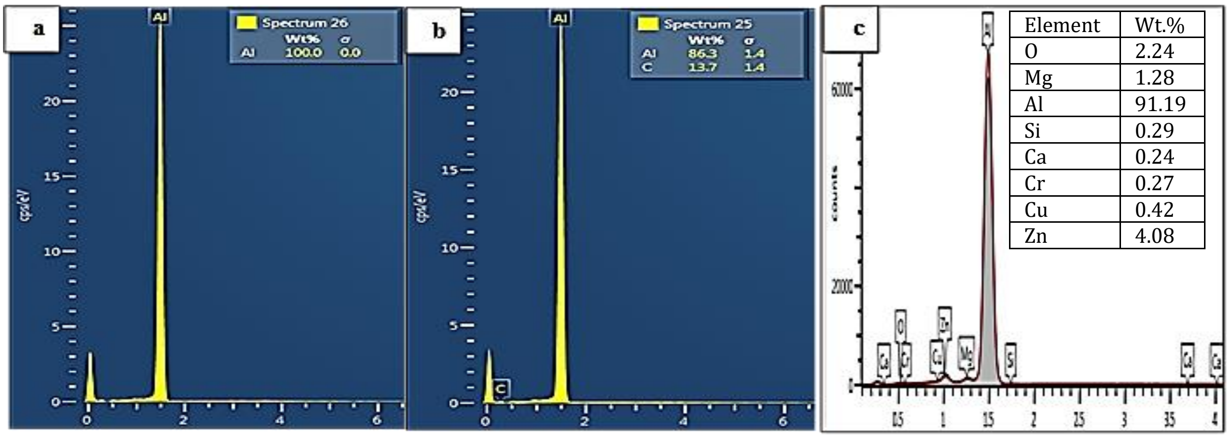

Figure 2. EDS of prepared samples; a) Al, b) Al-MWCNTs and c) Al7020 alloy.

2.2 Casting

Pure aluminium and Al7020 alloy were melted in a graphite crucible separately using gas-fired open furnace (see Figure 3a). The melt was superheated to 1000°C with flex materials (or/and 1 wt% CNTs powder put on closed aluminium foil and charged through the molten). Stainless steel impeller blade was used for mechanical stirring [14]. The melt was then cast into pre-heated (400°C) stainless steel mould of the required dimensions (see Figure 3b). Sample of transverse vibration and mechanical test were machined from the cast bars. Al7020 alloy is found to be the easiest one in machining with finest discontinued chips followed by pure aluminium. While MWCNTs/Al composite exhibit difficult machinability and produced long chips and suffer from high ductility during machining.

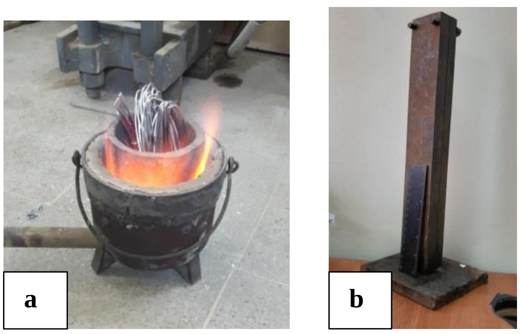

Figure 3. a) gas fired open furnace; b) casting mold.

2.3 Surface morphology

Scanning electron microscope (SEM) image was also taken for the samples using VEGA3 SCAN with electron voltage 20kV and magnification 1.0kX. Olympus microscope was utilized to determine the sample morphology using etching solution (applying cotton swab in the mixture then swapped the very well pre-polished surface for about 10 s, then rinsed and dried to examine [22]. Aluminium alloys and composite responded faster than pure aluminium).

2.4 Transvers Vibration Test

TQ Universal Vibration Apparatus TM16 (see Figure 4) was utilized to investigate the (30 × 1.25 × 1.20) mm^3 bar samples. Loaded (1.6 and 3.6 kg) and unloaded samples were tested with different frequencies (0-50 Hz). The data was collected and the relationship between amplitude and frequency was plotted.

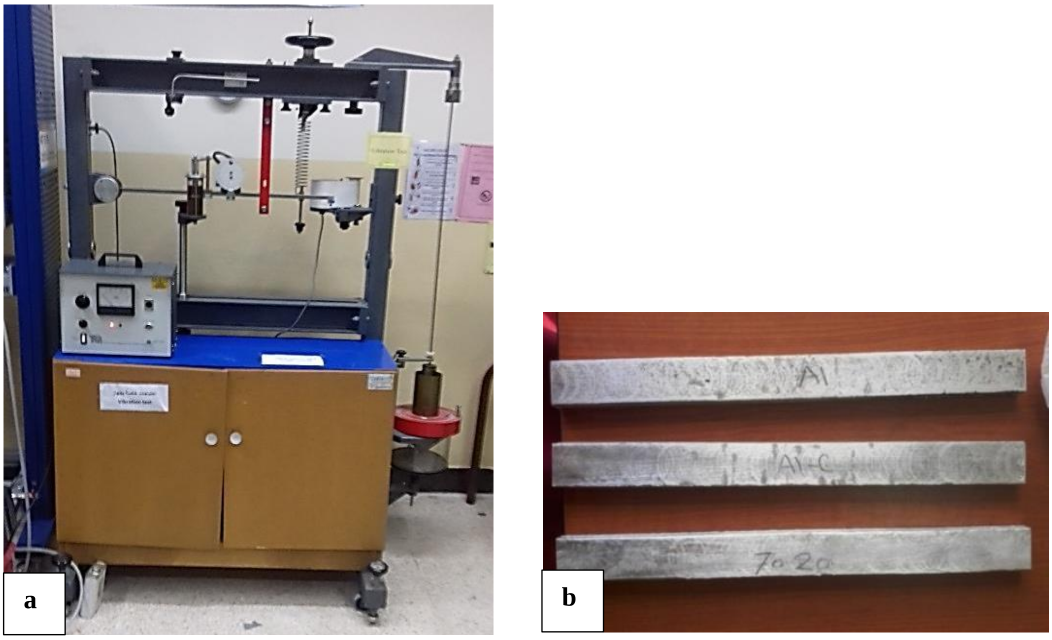

Figure 4. a) TQ Universal Vibration Apparatus (TM16); b) vibration samples.

2.5 Mechanical Tests

Tensile and three points bending tests were executed to the prepared samples according to ASTM E8 and ASTM E290 respectively using Wp 300 universal material tester [23,24]. The tests were executed at room temperature. The ductility of the composites was determined as percentage elongation. After successful testing, samples' hardness was also determined utilizing Vickers hardness apparatuses according to ASTM E18 [25]. Figure 5 shows the samples before and after testing.

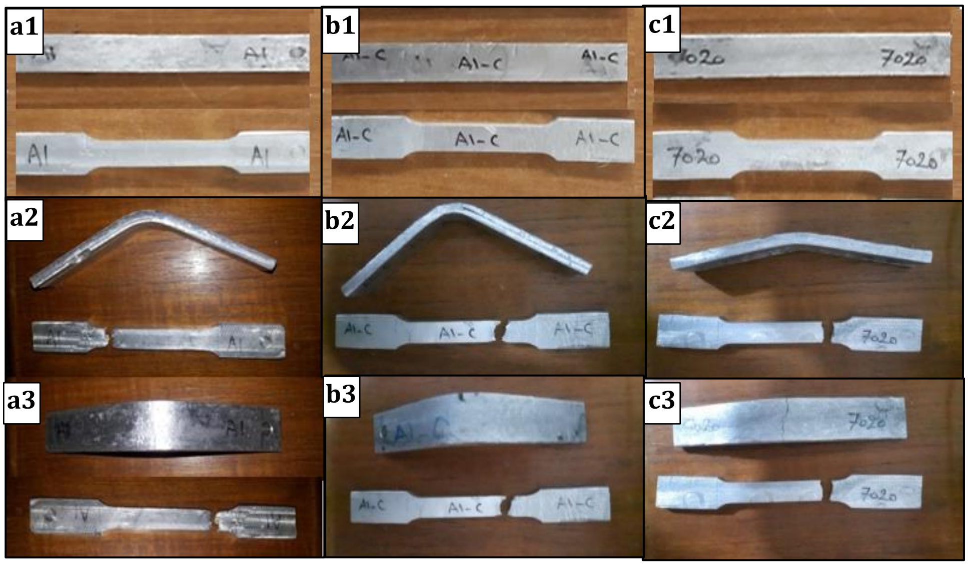

Figure 5. Tensile and bending samples; a,b,c) Al, Al-MWCNTs, Al7020 alloy respectively, a1,b1,c1) Al, Al-MWCNTs, Al7020 alloy before testing; a2,b2,c2) after testing (side view), a3,b3,c3) after testing (top view).

3. RESULTS AND DISCUSSION

Surface morphology of prepared samples was investigated by SEM with EDS and optical microscope. Figure 6 (a1,b1,c1) shows the SEM images of Al, Al7020 and Al/MWCNTs composites respectively. Figure 6 (a2,b2,c2) shows the optical microscopic of the same samples. Different spectrum spot was taken to analyze the surface chemically by EDS; Table 2 and 3 illustrate the results of EDS for Al7020 and Al/MWCNTs composite respectively. Variety of microstructure of Al, alloy and composite were noticed. The pure aluminium surface revealed the grain boundaries and pitting corrosion groves due to the etching solution and with no traces of phases. While the Al7020 topography showed the other phases due to alloying the aluminium with other elements (Zn, Mg, Cu, Si, Cr, Ca and others). These phases acquire materials the stiffness and the hardness when impeding the dislocations to move [26]. The SEM and optical microscope images of Al/MWCNTs composite showed the homogenous distribution of MWCNTs through the Al matrix grain boundaries which act as self-lubricant materials leading to increasing the ductility, increment of stiffness and hardness [27].

Table 2 EDS results of Al7020 alloy

Result Type	Weight %								
Spectrum Label	**O**	**Mg**	**Al**	**Si**	**Ca**	**Cr**	**Cu**	**Zn**	**Total**
Spectrum 1	2.24	1.28	91.19	0.29	0.24	0.27	0.42	4.08	100.00
Spectrum 2	1.36	0.77	94.31	0.32	0.01	0.32	0.20	2.72	100.00

Table 3 EDS results of Al/MWCNTs composite

Result Type	Weight %	
Spectrum Label	**Al**	**C**
Spectrum 1	81.14	18.60
Spectrum 2	77.76	22.24
Spectrum 3	100	0

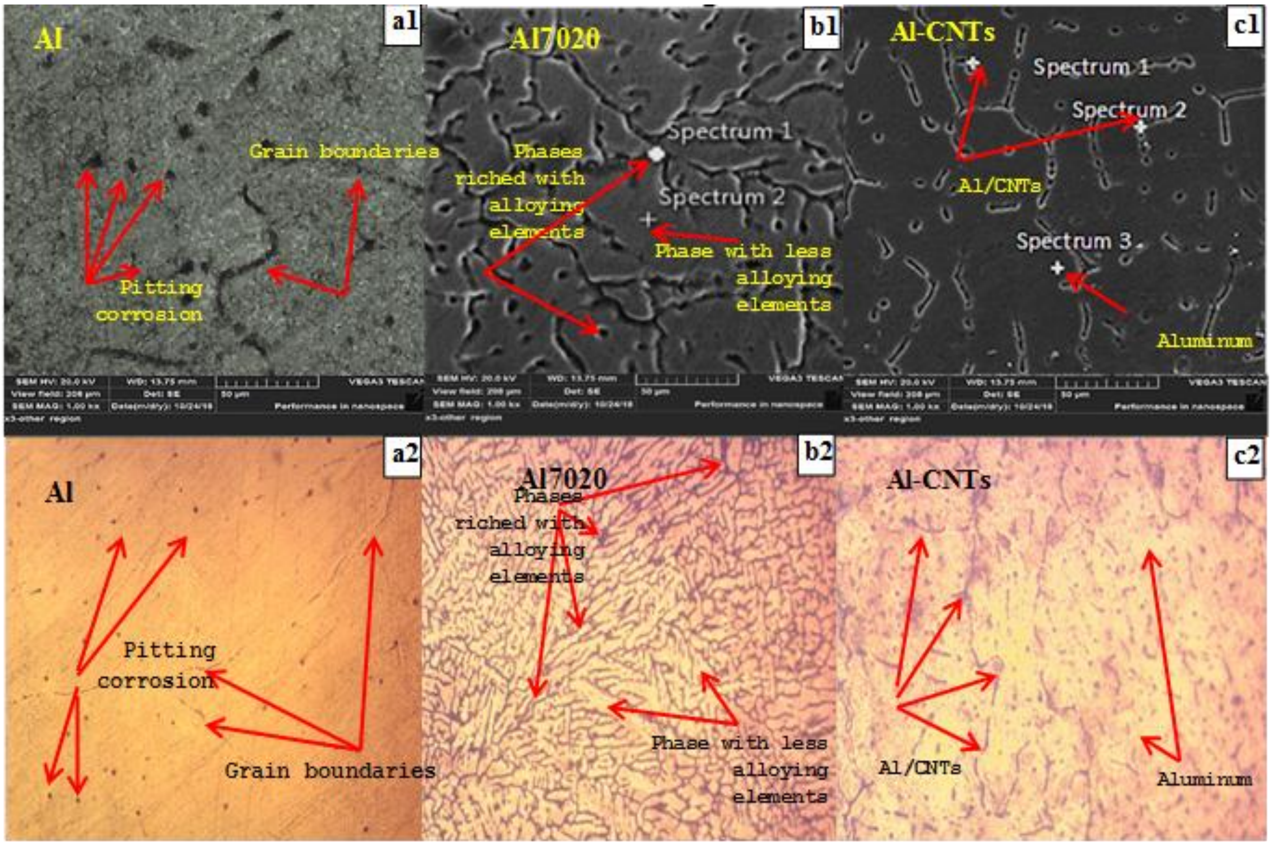

Figure 6. SEM and optical microstructure a,b,c) Al, Al-MWCNTs, Al7020 alloy respectively, a1,b1,c1) SEM images; a2,b2,c2) Optical microscopic images.

From tensile test results, stress-strain curves were plotted (see Figure 7 a,b,c). It can be observed that the tensile strength of Al7020 was enhanced (229 MPa) in comparison with pure aluminium (66 MPa). The standard Al7020 alloy has ultimate tensile strength and elongation percentage at thickness ≤100 mm is about ≥ 280MPa and ≥ 10% respectively, which decreased when casting and found to have is 229MPa and 8.8% due to the samples processing conditions (casting and machining) differs from ideal conditions. Whereas, the addition of 1wt% of MWCNTs to the aluminium matrix, the ultimate tensile strength of the composite decremented by 50% relative with the matrix. Therefore, the composite became lower stiff (see Table 4) due to the carbon present in the composite. This table illustrates the values of the tensile properties of the prepared samples. It can be observed that Al7020 yield before pure Al and Al-MWCNTs composites and showed a very narrow elastic region compared with the plastic deformation region, followed by pure Al and finally Al-MWCNTs composite. Failure surface of Al7020 behaved as a brittle fracture with no necking stage whereas the Al and composite have a ductile fracture. The ductility was also decremented for alloy and composites relative with pure aluminium due to the materials discontinuity. Aluminium stiffness was also affected by alloying and compositing, so the Al-MWCNTs revealed lower value than Al stiffness whereas Al7020 exhibited higher value than aluminium. These results agreed with [14] and disagreed with others [16,17,27] which they obtained hard aluminium carbide through Al/CNTs structure leads to increase the tensile strength of the Al matrix.

Table 4 Tensile test properties

Sample	Yield strength (MPa)	Ultimate tensile strength (MPa)	Young Modulus E(GPa)	Axial Stiffness K (kN/mm)	Elongation% at break
Al	16.9	66.2	3.8	4.1	11.4
Al-MWCNTs	17.0	32.6	3.6	3.4	7.5
7020Al	15.0	229.3	3.0	5.9	8.8

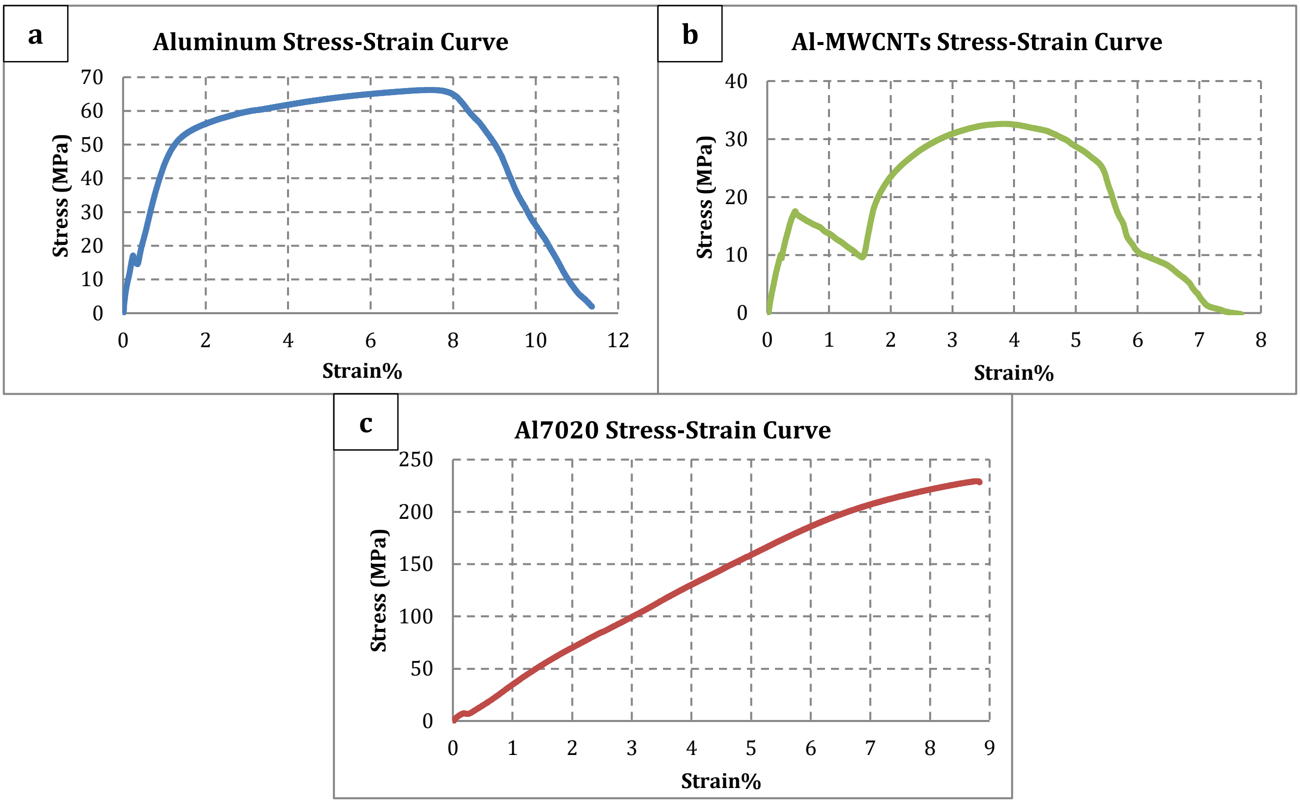

Figure 7. Tensile stress-strain curves for a) Aluminum, b) Al-MWCNTs, and c) Al7020. respectively.

Table 5 illustrated the bending properties of prepared Al, Al7020 and Al-MWCNTs composites. Figure 8 revealed the mechanical behaviour of materials under the bending force. From these results, the flexural strength and fracture toughness of aluminium base was enhanced when added with alloying elements (i.e. Al 7020) or by reinforcing materials (i.e. MWCNTs). While bending modulus of aluminium decremented when MWCNTs was added exhibiting maximum deflection under bending (73.498 %) so that CNTs acts as a self-lubricant material due to carbon layers (Graphene) slippage through grain boundaries. Al7020 showed other behaviour under bending, so the bending modulus incremented, while the deflection decremented as compare with aluminium base. This is due to the hard grain boundary phases which formed during solidification of alloying element with each other and/or aluminium base [26].

Table 5 Bending test properties

Sample	Flexural strength (MPa)	Bending Modulus E_b(GPa)	Fracture Toughness U (J)	Maximum Deflection%
Al	96	38.5	12.737	57.716
Al-MWCNTs	99	32.0	15.174	73.498
7020Al	430	42.0	23.583	24.886

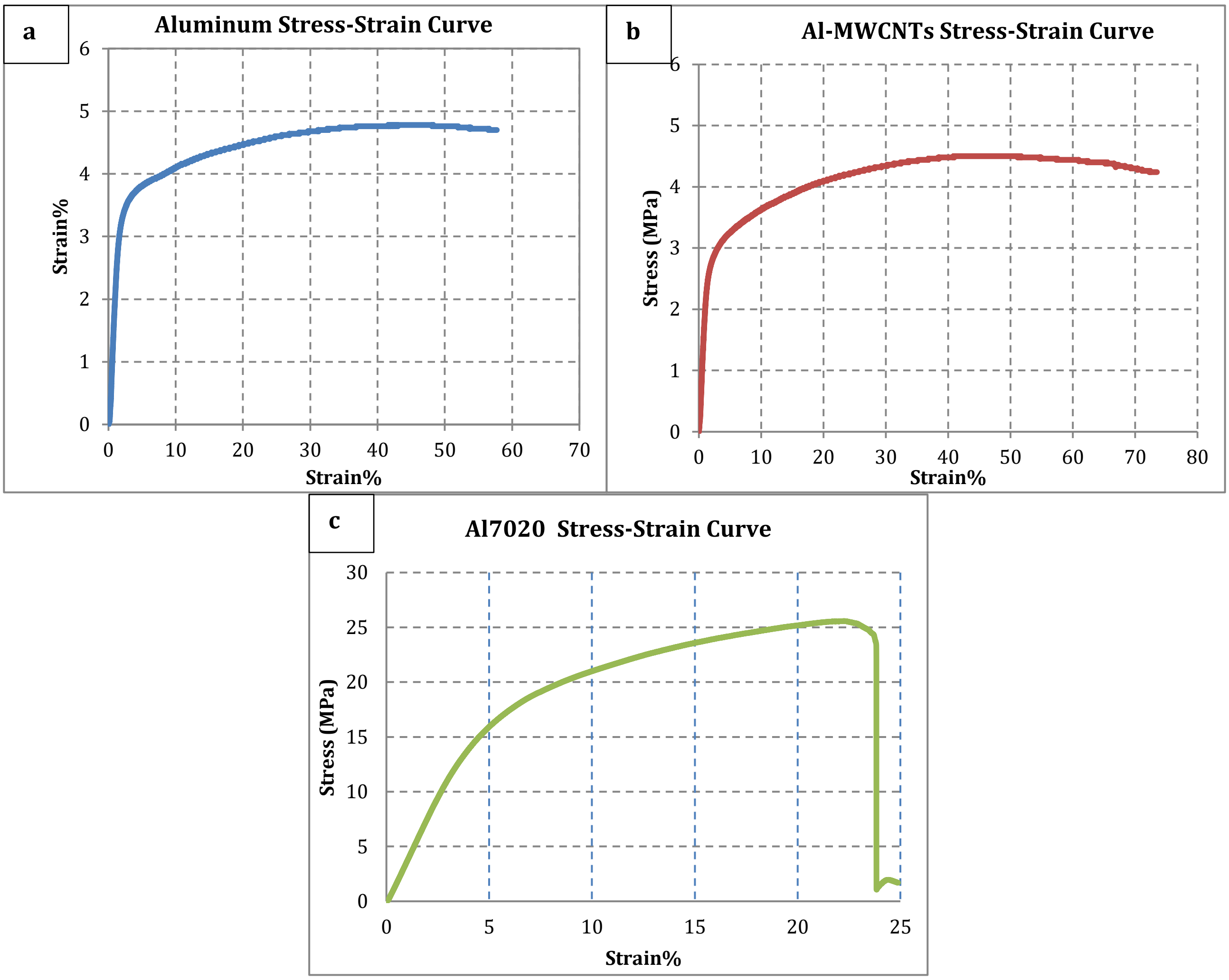

Figure 8. Three-points bending stress-strain curves for a) Aluminum, b) Al-MWCNTs, and c) Al7020.

The histogram in Figure 9 showed the Vickers hardness of the prepared samples. Alloying element addition to aluminium base (i.e. Al 7020) improves surface resistance to penetration by approximately 34%, which attributed to forming different phases hardened matrix impeding dislocation to move or slip. Whereas, MWCNTs addition to aluminium base deteriorates the surface hardness due to the easy slippage of carbon layers over each other's which agglomerated in grain boundaries encountering dislocations slippage.

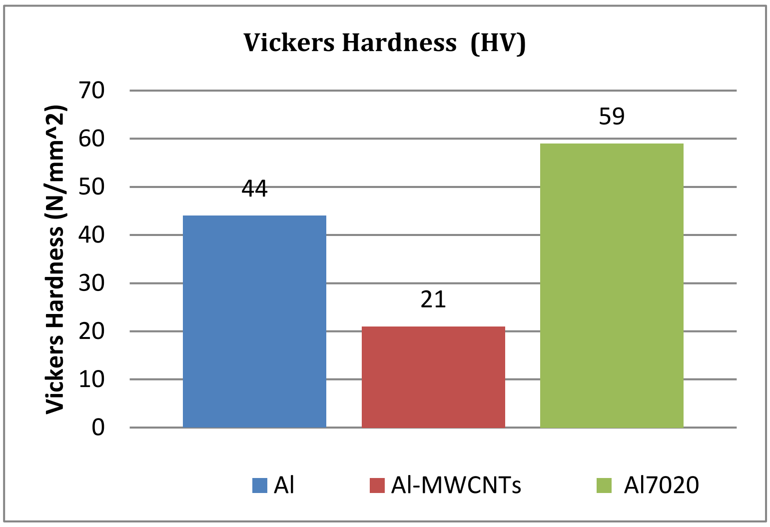

Figure 9. Vickers Hardness vs. mass of the prepared samples.

The response of the Al, A-MWCNTs and Al7020 under loading (1.6 and 3.2 kg) and unloading transverse vibration at different rotating speed (500-3000 rpm) using TQ Universal Vibration Apparatus (TM16) was plotted in Figure 10. It can be observed clearly that vibration response for each material is the difference for each other due to the stiffness and ductility effect on this response. This response was also changed with frequency and load. Thus, the pure aluminium at loaded and loaded cases showed the most damping effect (lowest amplitude fluctuating) along with the highest frequency range. But at maximum load (3.2 kg) and higher frequency (33.33-50 Hz) or (2000-3000 rpm), other behaviour was observed. Al-MWCNTs composites showed the most damping behaviour relative to Al and Al7020. Because at a higher frequency and maximum load, the material will vibrate with more stability and approaches to realty fluctuating. Al-MWCNTs composite has the lowest stiffness and hardness with higher deflection leading to better vibration damping.

According to these results, other experiments were executed to investigate the mass effect on vibration frequency. Figure 11 shows the plot between squared frequencies reciprocal versus mass (0-3200 g). From this figure, it is found that Al-MWCNTs composite exhibited the most damping effect along with mass ranges, then Al and Al7020 due to the lowest stiffness that is considered as the main material property which inversely affects the damping effect [28]:

$$\text{Natural frequency (Hz)} = 1/2\pi \sqrt{\frac{\text{Stiffness}}{\text{mass}}} \tag{1}$$

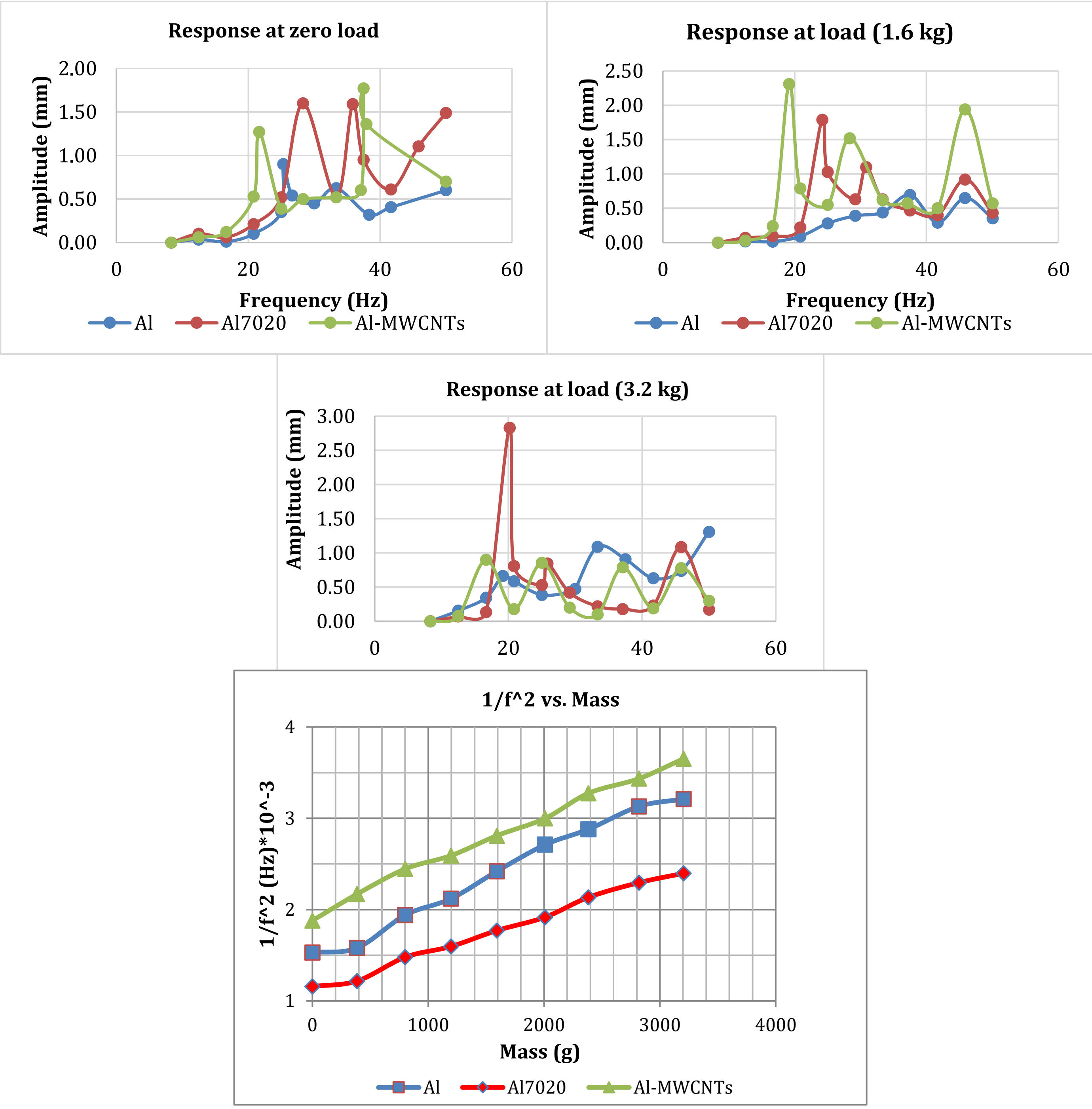

Figure 11. Reciprocal of squared frequency vs. mass of the prepared samples.

4. CONCLUSIONS

Aluminium, Al7020 alloy and Al-1wt% MWCNTs nanocomposites were prepared. Transverse vibration, mechanical characteristics and surface morphology were determined. The obtained results revealed different dynamic behaviour and mechanical properties of prepared samples. It was concluded that the different microstructure with different phases through matrix base has the main role of these variations. Alloying element in Al7020 alloy act as dislocations impeder make the alloy so hard, stiff and tough. While carbon nanotubes act as self-lubricant materials encourages the dislocations and defects to move easily. Due to this reason, the Al-MWCNTs nanocomposite showed minimum ultimate tensile strength, axial stiffness, and elongation percentage at break, bending modulus, fracture toughness with maximum deflection. Also, the dynamic behaviour of all samples is different under transverse vibration test when applied load and frequency were changed. The Al-MWCNTs nanocomposites exhibited the most stable structure under transverse vibration test at maximum applied load and higher frequencies.

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REFERENCES

[1] K. E. Drexler, "Engines of creation: The coming era of nanotechnology," New York, United States: Doubleday, (1986) 5-6.
[2] K. E. Drexler, "Nanosystems: Molecular Machinery, manufacturing, and computation," New York: John Wiley & Sons, (1992) 101-200.
[3] O. Kamigaito, "What can be improved by nanometer composites?," Journal of the Japan Society of Powder and Powder Metallurgy **38** (1991) 315–321.
[4] C. Lewitzke & P. Lee, "Application of elastomeric components for noise and vibration isolation in the automotive industry," SAE Technical Paper Series, Apr. 2001.
[5] M. Colakoglu, "Factors effecting internal damping in aluminum," Journal of theoretical and applied mechanics **42** (2004) 95-105.
[6] V. G. Geethamma, R. Asaletha, N. Kalarikkal, & S. Thomas, "Vibration and sound damping in polymers," Resonance **19** (2014) 821–833.
[7] S. H. Crandall, "The role of damping in vibration theory," Journal of Sound and Vibration, **11** (1970) 3–IN1.
[8] D. D. L. Chung, "Review: Materials for vibration damping," Journal of Materials Science **36** (2001) 5733–5737.
[9] K. U. Kainer, "Metal matrix composites: custom-made materials for automotive and aerospace engineering," John Wiley & Sons: New York, NY, USA, (2006) 145-150.
[10] P. Rohatgi, & B. Schultz, "Light weight metal matrix nanocomposites—Stretching the boundaries of metals," Mater. Lett. **2** (2007)16–19.
[11] Macke, B. F. Schultz, & P. Rohatgi, "Metal matrix composites offer the automotive industry an opportunity to reduce vehicle weight, improve performance," Adv. Mater. Process **170** (2012) 19–23.
[12] M. K. Surappa, "Aluminium matrix composites: Challenges and opportunities," Sadhana, **28** (2003) 319–334.
[13] F. Ostovan, K. Matori, M. Toozandehjani, A. Oskoueian, H. Yusoff, R. Yunus, and A. Mohamed Ariff, "Nanomechanical behavior of multi-walled carbon nanotubes particulate reinforced aluminum nanocomposites prepared by ball milling," Materials **9** (2016) 140.
[14] P. K. Rohatgi, N. Murali, H. R. Shetty, & R. Chandrashekhar, "Improved damping capacity and machinability of graphite particle-aluminum alloy composites," Materials Science and Engineering **26** (1976) 115–122.
[15] E. F. Crawley & M. C. Van Schoor, "Material Damping in Aluminum and Metal Matrix Composites," Journal of Composite Materials **21** (1987) 553–568.
[16] T. Kuzumaki, K. Miyazawa, H. Ichinose, & K. Ito, "Processing of Carbon Nanotube Reinforced Aluminum Composite," Journal of Materials Research **13** (1998) 2445–2449.

[17] C. F. Deng, D. Z. Wang, X. X. Zhang, & A. B. Li, "Processing and properties of carbon nanotubes reinforced aluminum composites," Materials Science and Engineering: A **444** (2007) 138–145.
[18] C. F. Deng, D. Z. Wang, X. X. Zhang, & Y. X. Ma, "Damping characteristics of carbon nanotube reinforced aluminum composite," Materials Letters **61** (2007) 3229–3231.
[19] M. Z. Hussain, U. Khan, A. K. Chanda, & R. Jangid, "Fabrication and Hardness Analysis of F-MWCNTs Reinforced Aluminium Nanocomposite," Procedia Engineering **173** (2017) 1611–1618.
[20] W. Zhou, T. Yamaguchi, K. Kikuchi, N. Nomura, & A. Kawasaki, "Effectively enhanced load transfer by interfacial reactions in multi-walled carbon nanotube reinforced Al matrix composites," Acta Materialia **125** (2017) 369–376.
[21] M. Shafique & X. Luo, "Nanotechnology in Transportation Vehicles: An Overview of Its Applications, Environmental, Health and Safety Concerns," Materials **12** (2019) 2493.
[22] G. F. V. Voort, "ASM Handbook Volume 9: Metallography and Microstructures," ASM International (2004) 1058-1060.
[23] Test Methods for Tension Testing of Metallic Materials. ASTM International; Available from: http://dx.doi.org/10.1520/e0008_e0008m-15a
[24] Test Methods for Bend Testing of Material for Ductility. ASTM International; Available from: http://dx.doi.org/10.1520/e0290-97ar04
[25] Test Methods for Rockwell Hardness of Metallic Materials. ASTM International; Available from: http://dx.doi.org/10.1520/e0018-12
[26] R. Eivani, H. Ahmed, J. Zhou, and J. Duszczyk, "Evolution of grain boundary phases during the Homogenization of AA7020 Aluminum Alloy," Metallurgical and Materials Transactions A 40 (2009) 717–728.
[27] H. Kwon, D. H. Park, J. F. Silvain, and A. Kawasaki, "Investigation of carbon nanotube reinforced aluminum matrix composite materials," Composites Science and Technology **70** (2010) 546–550.
[28] S. S. Rao, "Mechanical vibrations," 5 Ed., NEW JERSY, USA: Prentice Hall, (2011) 158-168.