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Wake potential and induced forces for protons in solids

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Abstract

The wake potential associated with the passage of swift protons through a solid material using cylindrical coordinates is studied. Full results of the wake potential as a function of space when a proton moves parallel with and perpendicular to the surface are presented. Three different points are approach: (i) Dielectric analysis based on the damping, velocity and kind of material (ii) The influence of a damping on the wake potential and induced forces (stopping and lateral) (iii) The coordinates of the internal.

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1. Introduction

N. Bohr (1948) [1] treatise the penetration of atomic particles through matter, shown in Fig. (1), a positive charged particle (Z_1e) penetrates a material. Let's look at the electric field slowing down the projectile, such a field is the result of the polarizability of the material. The electrons of the atoms are slightly displaced into the 'wake' of the projectiles. This effect is strongest behind the projectile than in front of it, and consequently the projectile feels a retarding force (Z_1eE_{ind}), where E_{ind} is the induced electric field. The retarding force is induced since the field is due to the presence of the projectile [2].

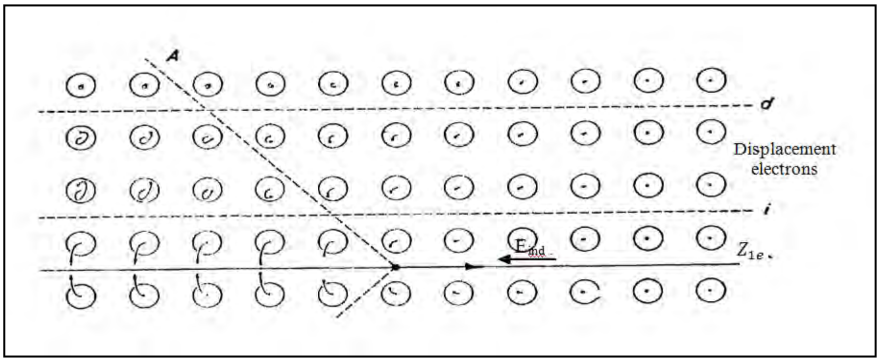

Fig. 1: The penetration of atomic particles through matter [1].

When the ion velocity is greater than the average velocity of valence electrons in solids, a good description of the loss process can be achieved using linear response theory,

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together with atomic-type calculations to evaluate the effect of losses due to core-electron excitations.

In condensed matter with many mobile electrons the charge of the ion becomes screening by the motion of the valence electron gas. The first nonlinear calculation in the static limit was performed by Echnique *et al.* [3] using the density functional formalism to calculate the response of the electron gas to the perturbation produced for a moving proton. These calculations have been extended to higher charges and were able to produce in a very natural way the oscillations of the stopping power, or equivalently the effective charge, as a function of the ion nuclear charge Z_1.

In this work a basic quantities that characterize the stopping of a charge interacting with a polarizable medium using semi classical arguments are introduced. The standard random phase approximation (RPA) representation for the dielectric function is used and several limiting analytical approximations are summarized [3].

2. Theory

2.1 Wake Potential

The wake potential (i.e. the scalar electric potential $\Phi_w(\vec{r})$) in a homogenous isotropic medium with dielectric properties $\varepsilon(\kappa,\omega)$ due to swift proton having a velocity $\vec{\upsilon}$ is given by [4]:

$$\phi_w(z,\rho) = \tfrac{2}{\pi\upsilon}\int_0^\infty q j_0(q\rho)dq \int_0^\infty \frac{d\omega}{q^2+\omega^2/\upsilon^2}\,\mathrm{Re}\left\{e^{i\omega z/\upsilon}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]\right\}, \qquad (1)$$

Where j_0 is the Bessel function of zero order.

The cylindrical coordinates ρ and z refers to the direction of motion, $\rho = x^2 + y^2$.

The wave number $\kappa^2 = q^2 + \dfrac{\omega^2}{\upsilon^2}$ has a component in $q-$and in the ρ-direction, where $q^2 = \kappa^2 - \omega^2/\upsilon^2$, thus,

$$dq = \frac{\kappa d\kappa}{\sqrt{\kappa^2 - \omega^2/\upsilon^2}} \qquad (2)$$

and Eq. (1) becomes,

$$\phi_w(r,\rho) = \frac{2Z_1^2}{\pi\upsilon}\int_0^\infty \kappa d\kappa j_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\int_0^\infty \frac{d\omega}{\kappa^2}\,\mathrm{Re}\left\{e^{i\frac{\omega z}{\upsilon}}\left[\frac{1}{\varepsilon(\kappa,\upsilon)}-1\right]\right\},$$

$$= \frac{2}{\pi\upsilon}\int_0^\infty \frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\int_0^\infty d\omega\,\mathrm{Re}\left\{e^{i\frac{\omega z}{\upsilon}}\left[\frac{1}{\varepsilon(\kappa,\upsilon)}-1\right]\right\} \qquad (3)$$

Because $[\frac{1}{\varepsilon(\kappa,\omega)}-1]$ is a complex function therefore Eq.(1) becomes,

$$\phi_w(z,\rho)=\frac{2Z_1^2}{\pi\upsilon}\int_0^\infty \frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\int_0^{\kappa\upsilon} d\omega$$

$$\text{X}\quad \left[\cos\left(\frac{\omega z}{\upsilon}\right)\text{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]-\sin\left(\frac{\omega z}{\upsilon}\right)\text{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]\right] \tag{4}$$

It's clear that Esq. (4) is strongly dependents on the dielectric analysis of the material and the internal coordinates z and ρ.

2.2 Stopping and Lateral Forces

The stopping power may be understood as a time-average force on the projectile, directed opposite to the velocity and originating in the response of the stopping medium to the electric field set up by the projectile. The response of the medium also includes a force component perpendicular to the direction of motion which contributes to lateral scattering of the projectile. For penetration in the bulk there is no net deflection because of symmetry. Therefore the lateral force only contributes fluctuating angular deflections (multiple scattering) [5].

The stopping force reflects the response of target electrons to the electric field induced by the projectile. The same is true for the lateral force which is known to be related to the electrostatic image force, and its significance in grazing-incidence studies with ion beams has been pointed out [6].

The induced electric field (i.e. the induced force) is given by the following Esq.:

$$F_{ind}(r)=-\frac{\partial\phi_w(r)}{\partial r} \tag{5}$$

Which may break down into its parallel and perpendicular components (i.e. z and ρ-components) with respect to the direction of motion of the proton, this field takes the following form:

(i) Stopping force, $F_z(z,\rho)=-\frac{\partial\phi_w}{\partial z}$,

$$F_z(z,\rho)=-\frac{2Z_1^2}{\pi\upsilon^2}\int_0^\infty \frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon}\omega d\omega J_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$

$$\text{X}\quad \left\{\sin\left(\frac{\omega z}{\upsilon}\right)\text{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]+\cos\left(\frac{\omega z}{\upsilon}\right)\text{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]\right\} \tag{6}$$

The stopping power (or average energy loss per unit path length $s_p = dE/dx$ is determined by the retarding force acting on the moving proton, which in this formulation is directly given by the value of induced electric field at the instantaneous position of the proton namely,

$$s_p = -F_z(0,0)$$

In other word,

$$s_p = \frac{2}{\pi\upsilon^2}\int_0^\infty \frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon}\omega d\omega\, \mathrm{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right] \qquad (7)$$

(ii) Lateral Force, $F_\rho(z,\rho) = -\frac{\partial\phi_w}{\partial\rho}$,

$$F_\rho(z,\rho) = -\frac{2Z_1^2}{\pi\upsilon}\int_0^\infty \frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon} d\omega J_1\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$
$$\mathrm{X}\left\{\cos\left(\frac{\omega z}{\upsilon}\right)\mathrm{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]-\sin\left(\frac{\omega z}{\upsilon}\right)\mathrm{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)}-1\right]\right\} \qquad (8)$$

Where $j_1(x)$ is the Bessel function of the first order. $F_\rho(0,0)=0$, because $j_1(0)=0$.

3. Dielectric Constant with No Damping, $\gamma \to 0$

3.1 Low ion velocity $\upsilon \le \upsilon_F$ (υ_F is Fermi velocity) in an electron gas:

The well-known Lindhard function [1954] [7] given in a self-consistent way an exact description of the dielectric function for a non-relativistic free electron gas of high density at zero temperature. In the low frequency limit, within this Random Phase Approximation (RPA) for the dielectric function, the loss function can be written as:

$$\varepsilon(\kappa,\omega) = \varepsilon_1(\kappa,\omega) + i\varepsilon_2(\kappa,\omega)$$

or $[\frac{1}{\varepsilon(\kappa,\omega)}-1] = \frac{1}{\varepsilon_1(\kappa,\omega)+\varepsilon_2(\kappa,\omega)}-1$

and for $\varepsilon_1(\kappa,\omega) >> \varepsilon_2(\kappa,\omega)$

$$[\frac{1}{\varepsilon(\kappa,\omega)}-1] = \frac{1}{\varepsilon_1(\kappa,\omega)+\varepsilon_2(\kappa,\omega)}-1 \cong \left[\frac{1}{\varepsilon_1(\kappa,\omega)}-1\right]-i\left[\frac{\varepsilon_2(\kappa,\omega)}{\varepsilon_1(\kappa,\omega)}\right] \qquad (9)$$

where ε_1 and ε_2 are the real and imaginary parts of the dielectric function.

$$\varepsilon_1(\kappa,\omega) = C(\kappa) f_1(\kappa) + 1 \qquad (10a)$$

$$\varepsilon_2(\kappa,\omega) = C(\kappa)\frac{\pi\omega}{2\kappa\kappa_F} \qquad (10b)$$

$$C(\kappa) = \frac{4\kappa_F}{2\kappa\kappa_F} \tag{10c}$$

$$f_1(\kappa) = \frac{1}{2}\left[1 + \frac{4\kappa_F^2 - \kappa^2}{4\kappa\kappa_F}\ln\left|\frac{\kappa + 2\kappa_F}{\kappa - 2\kappa_F}\right|\right] \tag{11}$$

Then,

$$\mathrm{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong \frac{1}{\varepsilon_1(\kappa,\omega)} - 1 = \frac{1}{C(\kappa)f_1(\kappa) + 1} - 1 \tag{12a}$$

$$\mathrm{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong -\frac{\varepsilon_2(\kappa,\omega)}{\varepsilon_1(\kappa,\omega)} \tag{12b}$$

Different approximations to the function $f_1(\kappa)$ lead to different expression for the wake potential and stopping and lateral forces [8].

If we take two approximations to $f_1(\kappa)$ as follows:

(i) $f_1(\kappa) = 1$, as a first approximation, then one can get the wake potential as follows:

$$\mathrm{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong \frac{C(\kappa)}{C(\kappa) + 1} = \frac{\left(\frac{4\kappa_F}{\pi\kappa^2}\right)}{\frac{4\kappa_F}{4\kappa^2} + 1} = \frac{\kappa_{TF}}{\kappa_{TF} + \kappa^2} \tag{13a}$$

$$\mathrm{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong -\frac{\varepsilon_2(\kappa,\omega)}{\varepsilon_1(\kappa,\omega)} = -\frac{2\kappa\omega}{\left[\kappa^2 + \kappa_{TF}^2\right]^2} \tag{13b}$$

where $\kappa_{TF} = \frac{4\kappa_F}{\pi}$ is Thomas Fermi wave number.

Therefore, the wake potential in Eq. (4) becomes,

$$\phi_w(z,\rho) = \frac{2Z_1^2}{\pi\upsilon}\int_0^{\infty}\frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2 - \omega^2/\upsilon^2}\right)\int_0^{\kappa\upsilon} d\omega$$

$$\mathrm{X}\ \left[\cos\left(\tfrac{\omega z}{\upsilon}\right)\left[\frac{\kappa_{TF}}{\kappa^2 + \kappa_{TF}^2}\right] + \sin\left(\tfrac{\omega z}{\upsilon}\right)\left[\frac{2\kappa\omega}{\left[\kappa^2 + \kappa_{TF}^2\right]^2}\right]\right], \tag{14}$$

The stopping force,

$$F_z(z,\rho) = \frac{-2Z_1^2}{\pi\upsilon^2}\int_0^\infty \frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon} \omega d\omega J_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$

$$\text{X} \quad \left\{\sin\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{\kappa_{TF}}{\kappa_{TF}^2+\kappa^2}\right] - \cos\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{2\kappa\omega}{\left[\kappa^2+\kappa_{TF}^2\right]^2}\right]\right\} \tag{15}$$

and the lateral force $F_\rho(z,\rho)$ in Eq. (8) becomes,

$$F_\rho(z,\rho) = -\frac{2Z_1^2}{\pi\upsilon}\int_0^\infty \frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon} d\omega J_1\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$

$$\text{X} \quad \left\{\cos\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{\kappa_{TF}}{\kappa_{TF}+\kappa^2}\right] + \sin\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{2\kappa\omega}{\left[\kappa^2+\kappa_{TF}^2\right]}\right]\right\} \tag{16}$$

At zero coordinates $\rho = 0$ and $z = 0$,

$$\phi_\omega = \frac{2Z_1^2}{\pi}\tan^{-1}\left(2\kappa_F/\kappa_{TF}\right) \tag{17}$$

And the stopping and lateral forces in Eqs. (15, 16) become,

$$F_z(0,0) = \frac{2Z_1^2\upsilon_1}{3\pi}\left[\ln\left(1+\pi k_F\right) - \frac{\pi k_F}{\left(1+\pi k_F\right)}\right] \tag{18}$$

From Brandt-Kitagawa (1982) [BK] we have, $\kappa_F = (9\pi/4)^{1/3}/r_s$ and $\alpha = (4/9\pi)^{1/3}$, with r_s is the radius of electron density.

$$\pi\kappa_F = \frac{\pi}{\alpha r_s} \tag{19}$$

Thus, Eq. (18):

$$F_z(0,0) = \frac{2Z_1^2\upsilon_1}{3\pi}\left[\ln\left(1+\frac{\pi}{\alpha r_s}\right) - \frac{1}{\left(1+\frac{\alpha r_s}{\pi}\right)}\right] \tag{20}$$

Let $Z = \dfrac{\pi}{\alpha r_s}$, then Esq. (20) becomes:

$$F_z(0,0) = \frac{2Z_1^2\upsilon_1}{3\pi}\left[\ln\left(1+Z\right) - \frac{Z}{\left(1+Z\right)}\right] = \frac{2Z_1^2\upsilon_1}{3\pi}I(Z) \tag{21}$$

where $I(Z) = \ln(1+Z) - \dfrac{Z}{1+Z}$ (22)

or

$$F_z(0,0) = \frac{2Z_1^2\upsilon}{3\pi} I(\pi\kappa_F)$$
(23)

which agree with Echnique (1980) [10].

The lateral force $F_\rho(0,0) = 0$, because the modified Bessel function of first order $J_1(0) = 0$.

(ii) The 2nd. Approximation to the function $f_1(\kappa)$ is obtained by using the full (RPA) dielectric response function, which has been proposed by Lindhard Winter (1964) [11]. Expanding the function $f_1(\kappa)$ up to the second order in k. one obtains:

$$f_1(k) = 1 - \frac{1}{3}\left(\frac{\kappa}{2\kappa_F}\right)^2 \qquad (24)$$

Using Esq. (12a, 12b) together with Esq. (24) one can get the following two Esq.:

$$\mathrm{Re}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong \frac{1}{(1 - A_2^2\kappa^2)} - \frac{A_3\kappa^2}{1 - A_2^2\kappa^2} \qquad (25)$$

$$\mathrm{Im}\left[\frac{1}{\varepsilon(\kappa,\omega)} - 1\right] \cong \frac{2\kappa\omega}{[B_2^2\kappa^2 + B_1^2]^2} \qquad (26)$$

wh13ere $A_1^2 = \left(\frac{1}{3} - \pi\kappa_F\right)$; $A_2 = \dfrac{A_1}{2\kappa_F}$ and $A_3 = \dfrac{1}{3}\left(\dfrac{1}{2\kappa_F}\right)^2$
(27)

and $B_1^2 = \dfrac{4\kappa_F}{\pi}$, $B_2^2 = 1 - \dfrac{1}{3\pi\kappa_F}$ (28)

The wake potential in Eq. (5) becomes:

$$\phi_w(z,\rho) = \frac{2}{\pi\upsilon}\int_0^\infty \frac{d\kappa}{\kappa} j_0\left(\rho\sqrt{\kappa^2 - \omega^2/\upsilon^2}\right)\int_0^{\kappa\upsilon} d\omega$$

$$\mathrm{X}\left[\cos\left(\tfrac{\omega z}{\upsilon}\right)\left[\frac{1}{1 - A_2^2\kappa^2} - \frac{A_3\kappa^2}{1 - A_2^2\kappa^2}\right] - \sin\left(\tfrac{\omega z}{\upsilon}\right)\left[\frac{2\kappa\omega}{[B_2\kappa^2 + B_1^2]^2}\right]\right] \qquad (29)$$

And the stopping and lateral forces in Esq. (7, 8) become:

$$F_z(z,\rho)=\frac{2z_1^2}{\pi\upsilon^2}\int_0^\infty\frac{d\kappa}{\kappa}\sqrt{\kappa^2-\omega^2/\upsilon^2}\int_0^{\kappa\upsilon}d\omega J_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$

$$\text{X}\quad\left\{\sin\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{1}{1-A_2^2\kappa^2}-\frac{A_3\kappa^2}{1-A_2^2\kappa^2}\right]+\cos\left(\tfrac{\omega z}{\vartheta}\right)\left[\frac{2\kappa\omega}{\left[B_2^2\kappa^2+B_1^2\right]^2}\right]\right\} \tag{30}$$

$$F_\rho(z,\rho)=-\frac{2}{\pi\upsilon}\int_0^\infty\frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon}d\omega J_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)$$

$$\text{X}\quad\left\{\cos\left(\frac{\omega z}{\upsilon}\right)\left[\frac{1}{1-A_2^2\kappa^2}-\frac{A_3\kappa^2}{1-A_2^2\kappa^2}\right]-\sin\left(\frac{\omega z}{\upsilon}\right)\left[\frac{2\kappa\omega}{\left[B_2^2\kappa^2+B_1^2\right]^2}\right]\right\} \tag{31}$$

Again at zero coordinates i.e. $\rho=0$ and $z=0$, the wake potential and stopping and lateral forces in Eqs. (29, 30 and 31) become:

$$\phi_\omega(0,0)=\frac{2Z_1^2}{\pi}\left[\frac{-1}{A_2}\ln\tan^{-1}\left(\sec^{-1}\frac{A_2\kappa}{2}\right)+\frac{1}{A_2^3}\left[2A_2\kappa_F+\frac{1}{2}\ln\left(\frac{2A_2\kappa_F-1}{2A_2\kappa_F+1}\right)\right]\right] \tag{32}$$

$$F_z(0,0)=\frac{2Z_1^2\upsilon}{3\pi}\left[\ln\left[\frac{1+2\chi^2/3}{\chi^2}\right]-\frac{1-\chi^2/3}{1+\chi^2/3}\right]\left[\frac{1}{\left(1-\chi^2/3\right)^2}\right] \tag{33}$$

Where $\chi^2=1/\pi\kappa_F$

and the lateral force $F_\rho(0,0)=0$ because $J_1(0)=0$.

3.2 High ion velocity $\upsilon\geq\upsilon_F$ (υ_F is Fermi velocity) in an electron gas:

At high velocities, where the projectile can excite plasmons in the medium, using the (RPA) of the dielectric function which is given in the following Esq., [5]

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega(\omega+i\gamma)}\qquad,\qquad \nu\to 0 \tag{34}$$

The plasmon frequency is $\omega_p=\frac{3^{1/3}}{r_s^{3/2}}$ and the small constant γ represents damping processes. It follows that in the limit $\gamma\to 0$,

$$\text{Im}\left[\frac{1}{\varepsilon(\omega)}-1\right]=-\frac{\omega_p\pi}{2}\delta\left(\omega-\omega_p\right) \tag{35}$$

$$\mathrm{Re}\left[\frac{1}{\varepsilon(\omega)}-1\right]=\lim_{\zeta\to 0}\left[\frac{\omega_p^2\left(\omega^2-\omega_p^2\right)}{\left(\omega^2-\omega_p^2\right)^2+\zeta^2}\right]=\left[\frac{\omega_p^2}{\left(\omega^2-\omega_p^2\right)}\right] \tag{36}$$

By substituting Esq. (35, 36) into Esq. (5), one can get the wake potential at high velocity:

$$\phi_\omega(z,\rho)=\frac{2Z_1^2}{\pi\upsilon}\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\int_0^\infty d\omega\left[\cos\left(\frac{\omega z}{\upsilon}\right)\frac{\omega_p^2}{\omega^2-\omega_p^2}\right]$$
$$+\frac{Z_1^2\omega_p}{\upsilon}\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2-\omega_p^2/\upsilon^2}\right)\sin\left(\frac{\omega_p z}{\upsilon}\right) \tag{37}$$

The stopping force $F_z(z,\rho)$ is:

$$F_z(z,\rho)=-\frac{2Z_1^2}{\pi\upsilon^2}\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}\, j_0\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\int_0^\infty \omega d\omega\left[\sin\left(\frac{\omega z}{\upsilon}\right)\frac{\omega_p^2}{\omega^2-\omega_p^2}\right]$$
$$+\frac{Z_1^2\omega_p^2}{\upsilon^2}\ln\left(\frac{\kappa_+}{\kappa_-}\right)J_0\left(\rho\sqrt{\kappa^2-\omega_p^2/\upsilon^2}\right)\cos\left(\frac{\omega_p z}{\upsilon}\right) \tag{38}$$

And the laterals force $F_\rho(z,\rho)$: becomes,

$$F_\rho(z,\rho)=\frac{2Z_1^2}{\pi\upsilon}\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}\, j_1\left(\rho\sqrt{\kappa^2-\omega^2/\upsilon^2}\right)\sqrt{\kappa^2-\omega^2/\upsilon^2}\int_0^\infty d\omega$$
$$\mathrm{X}\left[\cos\left(\frac{\omega z}{\upsilon}\right)\frac{\omega_p^2}{\omega^2-\omega_p^2}\right]+\frac{Z_1^2\omega_p^2}{\upsilon^2}\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}J_1\left(\rho\sqrt{\kappa^2-\omega_p^2/\upsilon^2}\right)\sin\left(\frac{\omega_p z}{\upsilon}\right)$$
$$\mathrm{X}\ \sqrt{\kappa^2-\upsilon^2/\upsilon^2} \tag{39}$$

where κ_+ and κ_- are the upper and lower integration limits in k are maximum and minimum transfers κ_+ and κ_- to target electrons [12,13] .

$$\kappa_\pm=\left[2\left(\upsilon^2-\beta^2\right)\pm 2\left[\left(\upsilon^2-\beta^2\right)^2-\omega_p^2\right]^{1/2}\right]^{1/2} \tag{40}$$

with $\beta^2=\frac{3}{5}\kappa_F^2$ and $\kappa_F=1.919/r_s$.

At zero coordinates $z\to 0$ and $\rho\to 0$, the wake potential in Eq.(37) becomes:

$$\phi_\omega(0,0)=\left(\frac{2Z_1^2\omega_p}{\pi\upsilon}\right)\int_{\kappa_-}^{\kappa_+}\frac{d\kappa}{\kappa}\ln\left[\tan\left(\sec^{-1}\left(\kappa\upsilon/\omega_p\right)\right)/2\right] \tag{41}$$

the stopping force $F_z(0,0)$:

$$F_z(0,0) = -\frac{Z_1^2\omega_p^2}{\upsilon^2}\ln\left(\frac{\kappa_+}{\kappa_-}\right) \tag{42}$$

and the lateral force at zero coordinates $F_\rho(0,0) = 0$ because the Bessel function at first degree $J_1(0,0) = 0$.

4. Dielectric constant under damping $\gamma > 0$:

Estimates of the energy loss of slow protons interacting with solids may be made using a free electron-gas model to describe the electronic response of the solid [14]. Earlier calculations for electrons interacting with an electron gas [15] showed that significant changes occur in the mean free paths and energy losses of low-energy electrons when one includes damping in the electron gas to account, in a phenomenological fashion, for the decay of elementary excitations as must occur in real solids. The increase in energy loss rate (or decrease in mean free path) is due to the possibility of plasmon excitation for electron energies below the threshold predicted in the absence of damping. A similar situation should obtain for heavy charged particles and could have important implications for estimates of the energy deposited in.

4.1 Slow ions $\upsilon < \upsilon_F$:

For an electron gas described by a complex dielectric function $\in(\kappa,\omega)$, the energy loss per unit path length for a carbon of velocity υ_1 in the electron gas (or the stopping power of the electron gas) is given by the Eq. (15), and a dielectric function approximation for the case of slow ions has been suggested by Ferrell et al [16]. They employ an approximate form for $\in(\kappa,\omega)$, the dielectric function of the metal, which is appropriate when energy transfer, ω, is small compared with the Fermi energy of the metal.

$$\in(k,\omega) = 1 + \omega_p^2 / \left\{s^2k^2\left[1 - i\pi\omega\theta(2k_F - k)/2k\upsilon_F\right] - \omega(\omega + i\gamma)\right\} \tag{43}$$

Eq. (43) is a simple generalization of the longitudinal dielectric function of an electron gas as derived from the hydro dynamical model [16], in which the prorogation velocity $s = \upsilon_F/\sqrt{3}$. The term proportional to ω multiplying s^2 k^2 in the denominator describes damping due to electron-hole excitation. It is chosen so that when equation (16) is expanded in a power series in ω it agrees with the small–ω expansion of $\in_{k,\omega}$,the Lindhard dielectric function [11], to first order. The presence of the factor $\theta(2k_F - k)$ accounts for the fact that $\mathrm{Im}\left(\frac{1}{\in_{k,\omega}}\right)$ vanishes when $k>2k_F$, since particle–hole excitations of small energy cannot correspond to a momentum transfer much greater than $2k_F$. The term containing γ as a factor describes “frictional” damping of collective states and may be taken from experiments for a given metal.

By substituting the imaginary part, $\mathrm{Im}\left(\frac{-1}{\in(\mathrm{k},\omega}\right)$ into Esq.(15) one can get :

$$S=\frac{2z_1^2}{\pi\,\upsilon_1^2}\int_o^\infty\frac{dk}{k}\int_0^{k\upsilon_1}d\omega\omega\,\mathrm{Im}\left[\frac{-1}{1+\omega_p^2/\left\{s^2k^2\left[1-i\pi\omega\theta(2k_F-k)/2k\upsilon_F\,\right]-\omega(\omega+i\gamma)\right\}}\right] \tag{44}$$

4.2 Swift ions $\upsilon>\upsilon_F$:

The interaction of fast ions with an electron gas is a problem of continuing interest. specifically, a great deal of theoretical and experimental work has been concerned with the distribution in space and time of perturbation of electron motion in solids caused by the passage of swift heavy charged particles. The explicit expression for the stopping power of a single charged particle was given by Eq.(15), [12,13].

By substituting the imaginary part of $\left(\frac{1}{\in(\mathrm{k},\omega)}\right)$ for a suitable approximation of the dielectric response ∈(k,ω) in the limit of damping process in Eq.(15), the stopping power can be obtained.

4.3 Random phase approximation (RPA) dielectric response function

One can represent the dielectric response function of the medium by RPA, Eq(34), with damping

$$\mathrm{Im}\left(\frac{-1}{\in(k,\omega)}\right)=\frac{\omega\gamma\left[\left(\omega^2-\omega^2+\omega_p^2\right)\right]}{(\omega^2-\omega_p^2)^2+\omega^2\gamma^2}\ , \tag{45}$$

And Eq. (7), yields:

$$s_p=\frac{2}{\pi\upsilon^2}\int_0^\infty\frac{d\kappa}{\kappa}\int_0^{\kappa\upsilon}\omega d\omega\left[\frac{\omega\gamma\left[\left(\omega^2-\omega^2+\omega_p^2\right)\right]}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\gamma^2}\right] \tag{46}$$

Eq. 46 gives the stopping power at high velocity and under damping effect $(\gamma>0)$, and must be solved numerically. A program **WakePot.f90** has been written in FORTRAN-90 with aid of software 'Compaq Visual Fortran 6.6' for compiling, linking and executing the program.

5. Results and conclusions
5.1 Wake potential

The wake potential considered in the present work for the materials indicated before. The calculation of wake potential is according to Eq. (5) using RPA dielectric function taking in the consideration the special cases of velocity and influences of damping. Figs. 1(a-b) show the values of the wake potential calculated along the projectile trail (i.e at $\rho = 0$) and at $(\rho = 0.5, 1)$, for a proton moving with velocities $\upsilon = 0.4, 1, 12 a.u$ in gold, Au and aluminum, Al. The general shape of the wake potential derived from Eq.(5) shows a damped oscillatory behavior in the longitudinal direction behind the projectile; the pattern of these oscillations decreases exponentially in the transversal direction. Also, this wake potential extends slightly ahead of the projectile. The surface-wake potential in the points of trajectory, when the proton is moving to the right near an (gold or aluminum) surface, for different impact parameter and low velocities. A velocity behavior similar to that observed in the bulk wake is seen through the series of plot: for $\upsilon = 0$ the potential has a dip at the position of the particle; when the ion moves slowly $(\upsilon \leq \upsilon_F)$ the dip is shifted to the left; at large $(\upsilon \geq \upsilon_F)$ some oscillation appear behind the particle, whose wavelengths increase with the velocity according to $2\pi\upsilon/\omega_s$ in the solid side and $2\pi\upsilon/\omega_s$ in the vacuum [4].

Figs. 2(a-b) show the three-dimensions surface wake potential calculated in the plane containing both the surface normal and the particle trajectory for velocities $(\upsilon = 0.4, 1, 12 a.u)$ and different positions of the ion. A trough (hill) is observed at the variation of induced potential with axis ρ and z. When the particle follows an inner trajectory, but stays close enough to the surface $(|z| < \pi\upsilon/2\omega_s)$, both oscillations of frequency ω_s and ω_p coexist, as the ion travels deeper inside the solid, the latter becomes dominant (Fig.(2(a-b)).

Fig. 3(a-b) show the stopping force Fs in (eV/A) units as a function of z and at three different values of ρ (0, 0.5 and 1) and velocities υ (0.4, 1 and 12 a.u). The point (ρ =0, z=0) is the starting point of interacting protons with the target, in other word it is a projectile trail where the effective of the force is high, but when ρ increases the effective decrease, because the projectile far from the reference point (ρ =0, z=0).

Fig 4 (a-b) shows the three dimensions surface stopping force in the plane containing both the surface normal and the particle trajectory for velocities $(\upsilon = 0.4, 1, 12 a.u)$ at different positions of the ion.

The lateral force $F_L(\rho, z)$ at ρ =0 because the Bessel function of the first order $j_1(\rho, z) \rightarrow 0$ when $\rho \rightarrow 0$ as shown in Figs. 5(a-c). This means that there is no lateral force for the incident protons at parallel coordinates.

Finally Fig 5 (a-b) shows the three-dimension surface lateral force $F_L(\rho, z)$ in the plane at different positions of the ion ρ and z, and at different velocities $(\upsilon = 0.4, 1, 12 a.u)$.

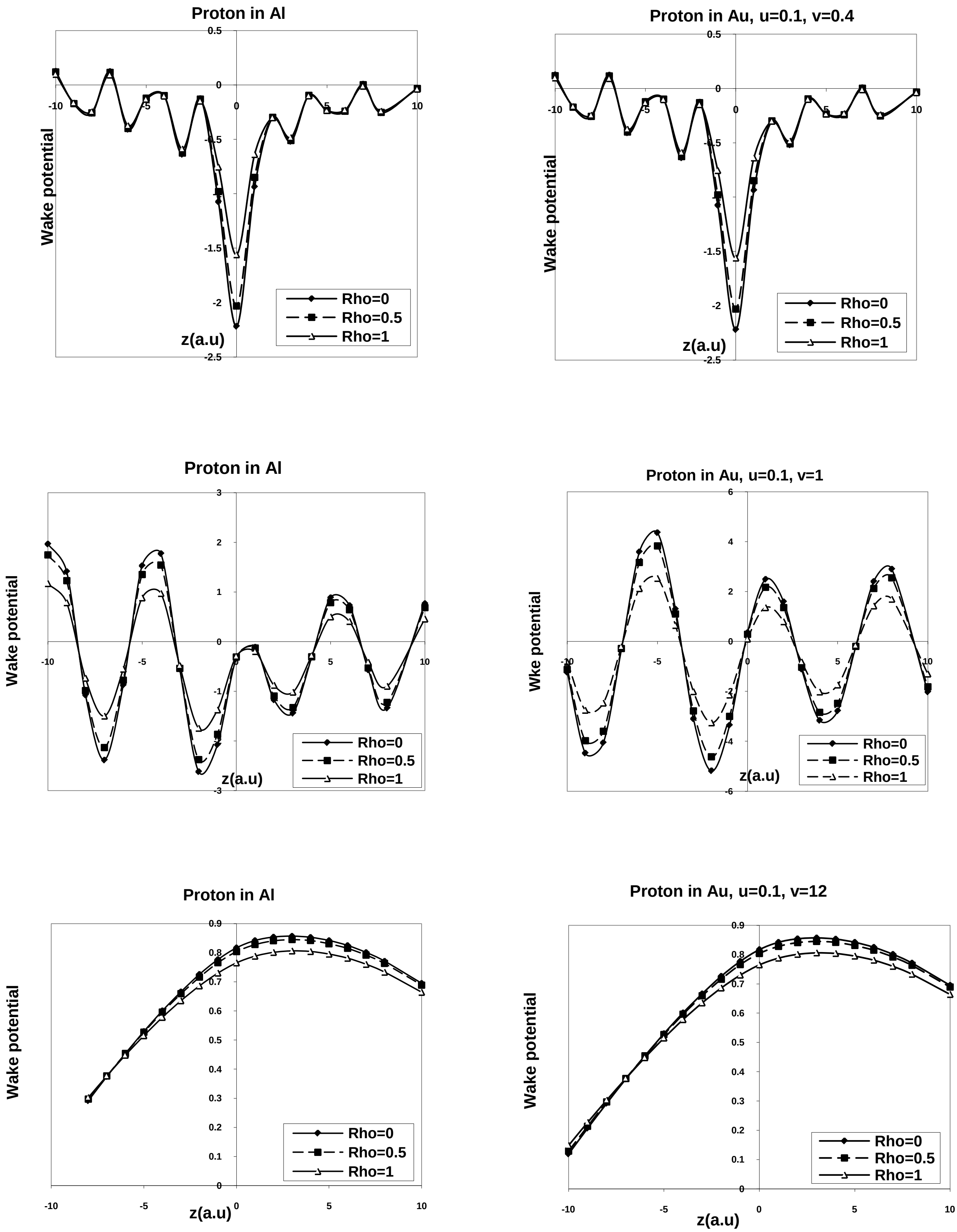
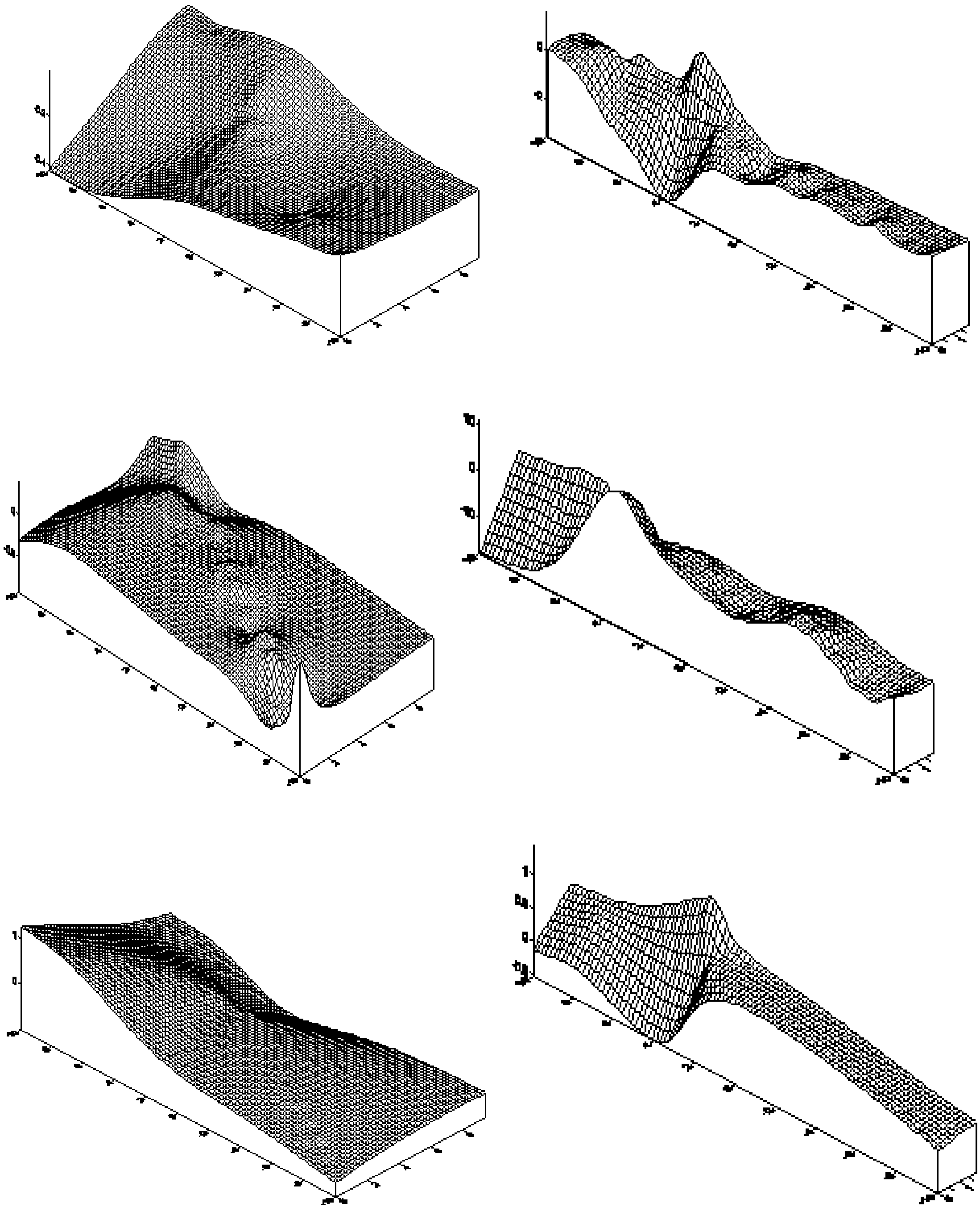
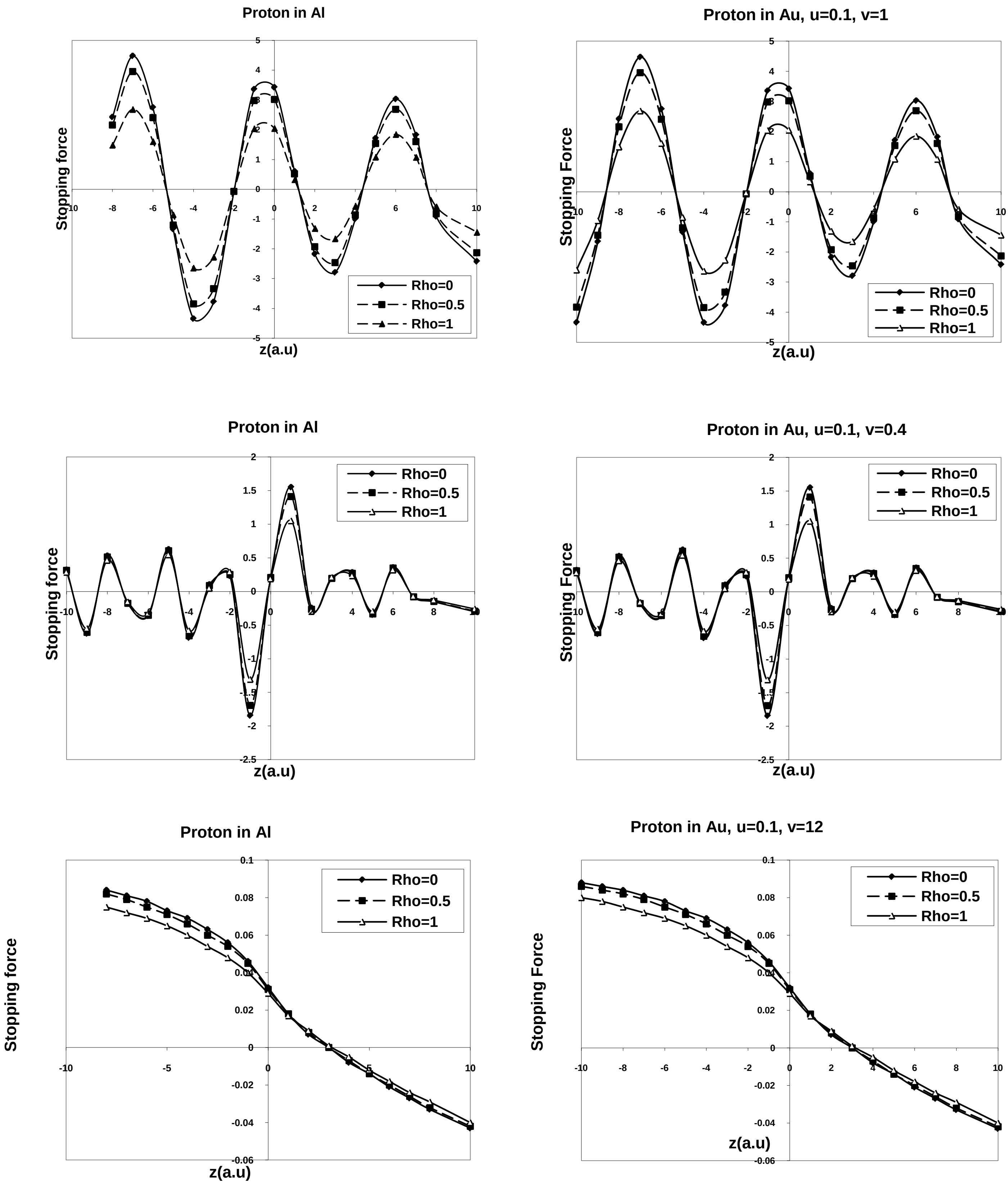


Fig. 2: The variation of stopping force with z (a.u), and $\rho(a.u)$ at damping u=0.1 and velocities $(0.4,1,12a.u)$ of proton in (a) Al and (b) Au.



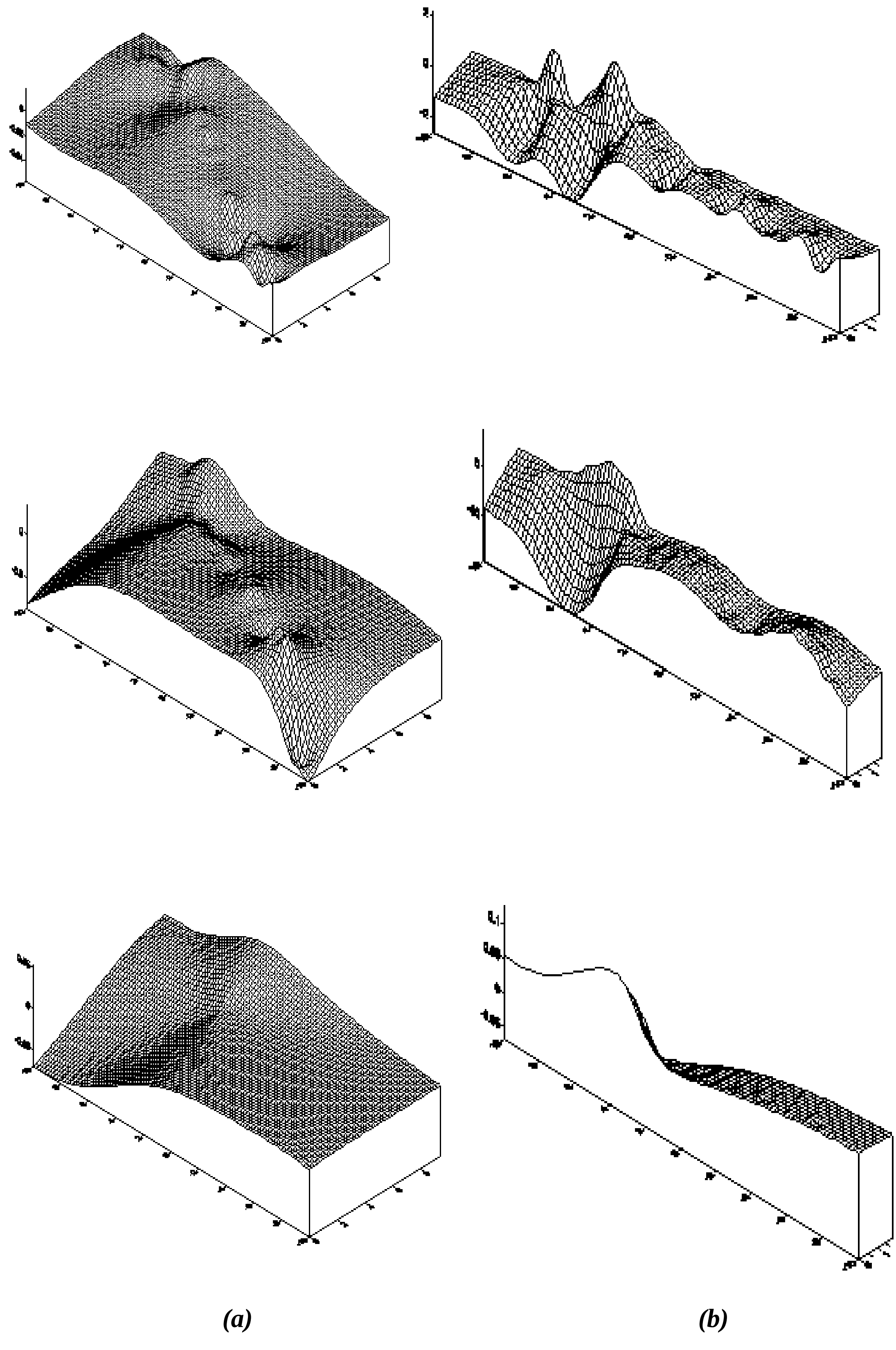
(a) *(b)*

Fig. 3: The surface wake potential at damping u=0.1, velocities $\upsilon = (0.4,1,12 a.u)$ with z (a.u) and $\rho(a.u)$ of proton in (a) Al and (b) Au.



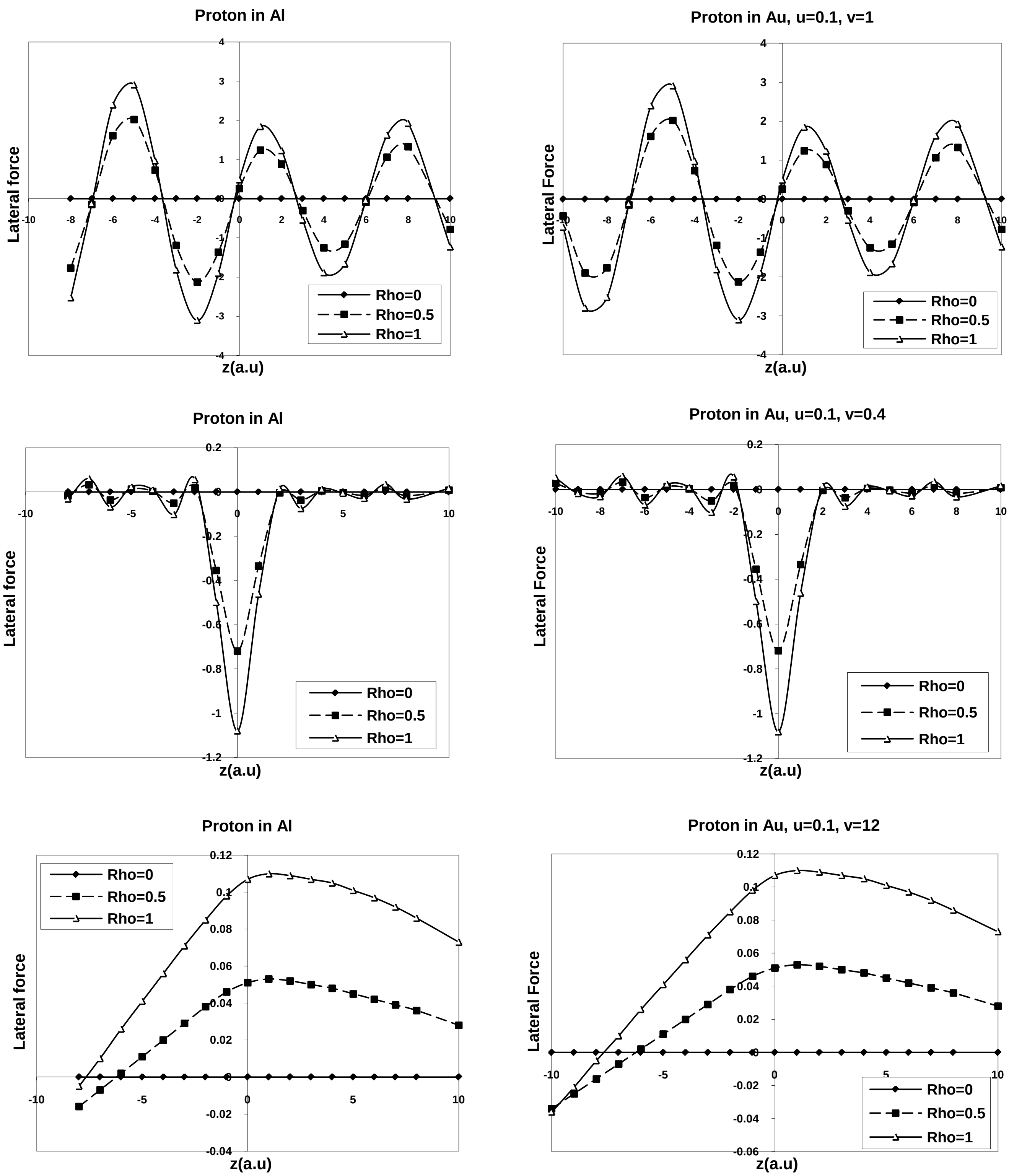
(a) ***(b)***

Fig. 4: The variation of stopping force with z (a.u), and $\rho(a.u)$ at damping u=0.1 and velocities $(0.4, 1, 2a.u)$ of proton in (a) Al and (b) Au.



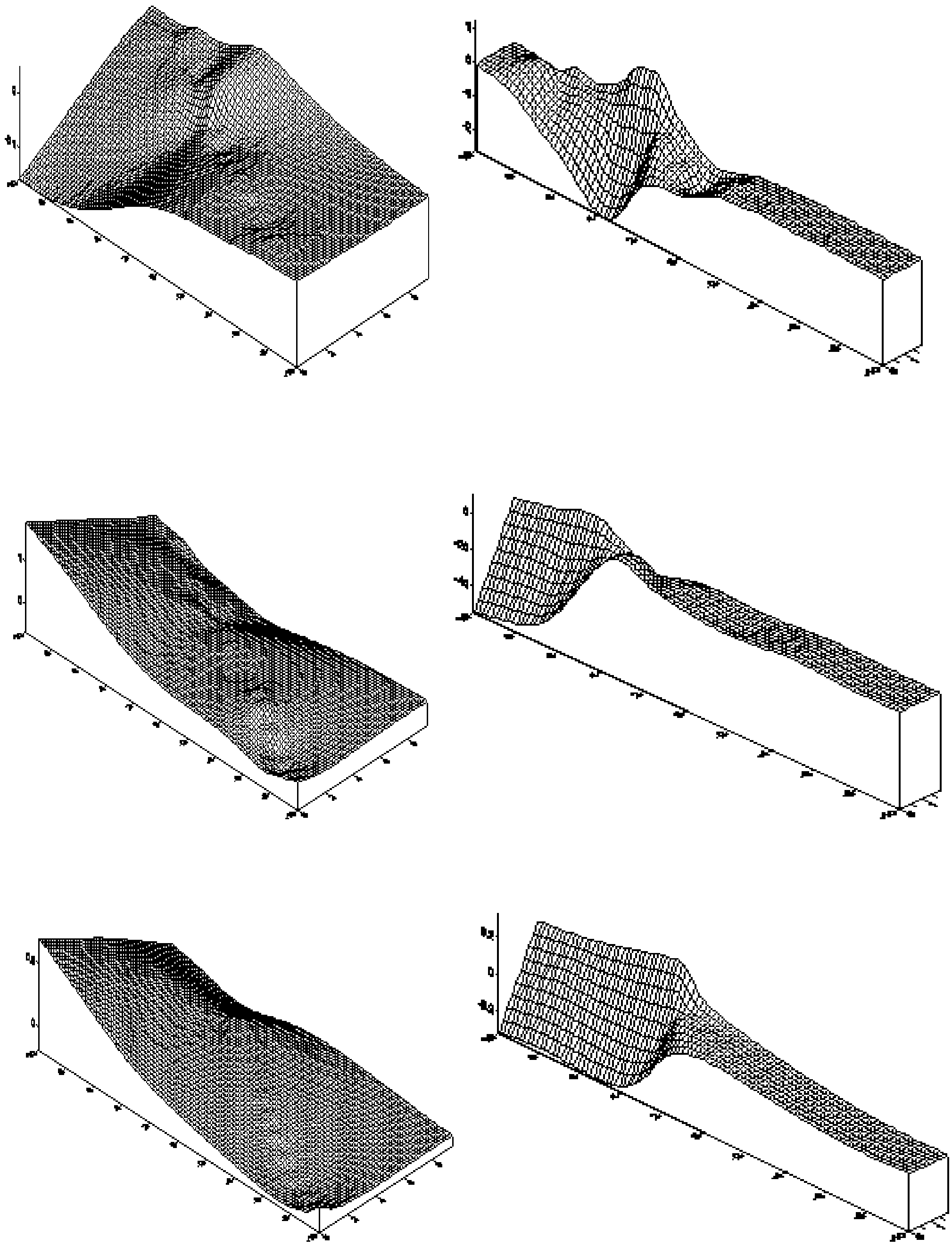
(a) *(b)*

Fig. 5: The surface stopping force at damping u=0.1, velocities $v = (0.4, 1, 2a.u)$ with z (a.u) and $\rho(a.u)$ of proton in (a) Al and (b) Au.



(a) *(b)*

Fig. 6: The variation of lateral force with z (a.u), and $\rho(a.u)$ *at damping u=0.1 and velocities* $(0.4, 1, 2a.u)$ *f* proton in (a) Al and (b) Au.



(a) *(b)*

Fig. 7: The surface lateral force at damping u=0.1, velocities $\upsilon = (0.4, 1, 2 a.u)$ with z (a.u) and $\rho(a.u)$ of proton in (a) Al and (b)Au.

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