

Optical Gain and Confinement in Gaas/Algaas Structure Quantum Well Lasers

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ABSTRACT

In this paper we investigate the optical gain seen by the polarization modes for each of Transverse Electric (TE) and Transverse Magnetic(TM) of GaAs/Al_{0.32}Ga_{0.68}As quantum-well lasers. The factor confinement and modal gain of proposed structure also have been simulated. Comparison between the two structures (single and multiple quantum) were conducted in the effort to evaluate and understand their behavior. Results show that the optical gain offers a better value in TE mode than in TM mode. As a result of being on higher standard and having a better performance in both cases thus this study conclude that the multiple quantum well structures are a better designed choice compared to single quantum well structure.

Keywords: Confinement factor, optical gain, polarization, quantum well laser.

1. INTRODUCTION

A great attention has been drawn on semiconductor quantum well (QW) structures due to the virtue of their novel electronic optical properties. The exceedingly high optical gain at very low current densities is a main property of QW. This occurs mostly because of the high carrier density in quantum well and partly because of greater population inversion at a given carrier density as a result of the lower quantized density of states. Generally, the optical gain of quantum well lasers is very high due to their high carrier confinement. The quantum well lasers optical confinement factor on the other side is relatively low on grounds of their thin active region. Quantum well lasers modal gain evaluation is needed to foresee the lasing behavior. The efficiency of injected carriers' collection and the optical confinement factor well determine the quantum well lasers' modal gain [2]. Investigations on GaAs/Al_xGa_{1-x}As quantum well structures with a thickness of 100Å have been widely reported [2, 3], although the use of thinner QW structures is relatively new. The importance of using QWs with polarization was shown, looking for improved optical gain and better confinement factor. These two factors are established for TE modes whereas TM mode is rarely described in literature. The aim of the present work is to calculate the optical gain in GaAs/Al_{0.32}Ga_{0.68}As quantum well laser using Lorentzian broadening function in Transverse Electric (TE) and Transverse Magnetic (TM) polarization modes. The factor confinement and modal gain also have been calculated. On the other hand, a comparative analysis was also carried out between single and multiple quantum well structures.

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2. CONDUCTION AND VALENCE BANDS' ENERGY LEVELS

Quantum well lasers optical gain is given by calculating the number of energy subband in quantum well and wave functions of electron and hole in active region which are given by solving time-independent Schrodinger equation. In order to solve this latter, finite difference numerical technique has been chosen. Using this technique, the eigenvalue problem of Schrödinger can be transformed into a matrix eigenvalue problem. The Schrödinger equation in one dimension is given by [5]:

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) + V(x)\psi(x) = E\psi(x) \quad (1)$$

Where \hbar refers to Planck's constant divided by 2π , and ψ to the particle wave function, V represents band potential and E to the energy level of the conduction or the valence bands and m is the effective mass of the carrier (electron/ hole) [6].

3. MATERIAL GAIN SPECTRUM

The proposed structure consists of a 60\AA thick p-GaAs active layer quantum well sandwiched between two 150\AA thick $n\text{-Al}_{0.32}\text{Ga}_{0.68}\text{As}$ cladding layers. The band-gap discontinuities at QW interfaces are $\Delta E_C = 240$ meV at conduction and $\Delta E_V = 160$ meV at valence bands respectively. The active region consists of five quantum wells which is 60\AA thickness with $\text{Al}_{0.32}\text{Ga}_{0.68}\text{As}$ barriers of the same thickness as a cladding layer in between in the multi-quantum well. A schematic band diagram of the GaAs/Al_{0.32}Ga_{0.68}As multiple quantum well laser structure is shown in Figure 1.

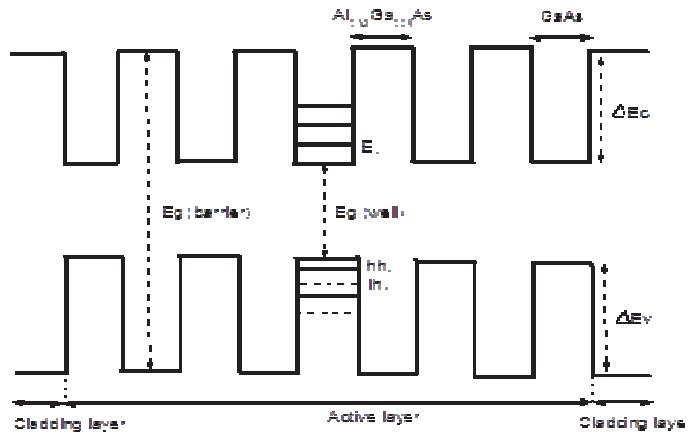


Figure 1. Schematic band diagram of a *GaAs/ Al_{0.32}Ga_{0.68}As* multiple quantum-well laser structure

As soon as MQW system is pumped, the carriers travel via barriers to drop in QW. Two quasi Fermi levels are formed as the excess carriers split the Fermi depending on the pumping strength. When the pumping strength is equal to the material band-gap, the material comes become transparent to photons with energies equal to the band-gap. As soon as the system pumping gets beyond transparency condition, the optical gain and transition that depends on the polarization as a function of photon energy was achieved and it is illustrated as in Eq. (2):

$$g(E') \frac{q^2 |M|^2}{E' \epsilon_0 m_0^2 c_0 \hbar n_{eff} L_z} \times \sum_{i,j} \int_{E_g}^{E_{gb}} m_{r,ij} C_{ij} A_{ij} (f_c - f_v) L(E' - E) dE \quad (2)$$

here q symbolizes the elementary charge of the electron, $|M|^2$ refers to the bulk momentum transition matrix element, and each of ϵ_0 and c refers to the permittivity of free space [2] and the light speed in vacuum respectively. n_{eff} is the laser structure effective refractive index, L_z is the QW thickness, ij are quantum numbers of conduction and valence band, $m_{r,ij}$ refers to the spatially weighted reduced mass for transition, C_{ij} symbolizes the spatial overlap factor between states i and j , A_{ij} symbolizes the angular anisotropy factor, and f_c and f_v are electron Fermi functions [8]. The parameter of the reduced mass is given by $m_{r,ij}^{-1} = m_i^{-1} + m_j^{-1} \cdot m_i$ and m_j here refers to weighted (by wave function confinement factor in QW) quantum well and cladding masses averages [9]. The spectral broadening of each transition used to be commonly included by lorentzian lineshape function in Eq. (3).

$$L(E' - E) = \frac{1}{\pi} \frac{\hbar/\tau_{in}}{(E' - E)^2 + (\hbar/\tau_{in})^2} \quad (3)$$

where τ_{in} refers to the relaxation intra-band time set at 2×10^{-13} s [10]. For TE transition, within the QW plane electric field vector, its values are $A_{ij} = (3/4)(1 + \cos^2 \theta_{ij})$ for (e-hh) transitions of electron-heavy hole and $A_{ij} = (1/4)(5 - 3\cos^2 \theta_{ij})$ for (e-lh) transitions of electron-light hole. For transition of transverse magnetic, within a normal electric field to the quantum well, its values are $A_{ij} = (3/2)(\sin^2 \theta_{ij})$ for e-hh transitions and $A_{ij} = (1/2)(4 - 3\sin 2\theta_{ij})$ for e-lh transitions. The angular factor is $\cos 2\theta_{ij} = E_{ij}/E$ [11]. When the energy of photons improved deeper into the band an improving anisotropy between transitions of heavy and light holes is indicated. Bulk averaged momentum matrix element in between valence and conduction states is:

$$|M|^2 = \frac{m_c^2 E_{go}(E_{go} + \Delta)}{6m_c(E_{go} + (E_{go} + 2\Delta/3))} \quad (4)$$

E_{go} is the direct band-gap calculated by the following relationship $E_{go} = [E_g - (5.405 \times 10^{-4} \times T^2)/(T + 204)]$. T is the operating temperature, E_g is the band-gap energy, Δ refers to the split-off band separation and m_c is the conduction band effective mass [12]. The occupation probability of the states in conduction and valance bands which is also known as Fermi function is given as follows:

$$f_{c,v}(E, E_{fc,fv}) = \frac{1}{1 + \exp[(E - E_{fc,fv})/kT]} \quad (5)$$

Where $E_{fc,fv}$ symbolizes the levels of quasi-Fermi in conduction and valance bands and k symbolizes the Boltzmann constant. The carrier density in a band is given by integrating the occupation probability distribution multiplied by a states' density over the entire band, the carrier density can be obtained for non-parabolic band structure, as:

$$n = \sum_n \int_0^{+\infty} \rho(k) [f_c(E_{cn}(k), E_{fc})] dk \quad (6)$$

$$P = \sum_{hh,lh} \int_0^{+\infty} \rho(k) [1 - f_v(E_{vn}(k), E_{fv})] dk \quad (7)$$

The summation includes all conduction (n) and valence ((hh , lh) bands states. The values of various physical parameters used in the simulation are displayed in Table 1.

Table 1 The used parameters' values

Parameter	Description	Value
C_0	<i>The light speed in vacuum</i>	3×10^8 [m/s]
Q	<i>The charge of electron</i>	1.6×10^{-19} [C]
\mathcal{E}_0	<i>The permittivity of free space</i>	8.854×10^{-12} [F/m]
h	<i>Planck constant</i>	6.626×10^{-34} [J.s]
K	<i>Boltzmann constant</i>	1.38×10^{-23} [J/K]
m_0	<i>Free electron mass</i>	0.91×10^{-30} [kg]
m_e	<i>Effective mass of electron</i>	$0.067 m_0$ (Ref. [13])
m_{hh}	<i>Effective mass of heavy hole</i>	$0.45 m_0$ (Ref. [13])
m_{lh}	<i>Effective mass of light hole</i>	$0.082 m_0$ (Ref. [13])
L_z	<i>Quantum well thickness</i>	60 [\AA]
Δ	<i>Spin orbit interaction energy</i>	0.34 [eV] (Ref. [13])
E_g (well)	<i>Well Band-gap energy</i>	1.43 [eV] (Ref. [14])
E_g (Barrier)	<i>Barrier Band-gap energy</i>	1.823 [eV] (Ref. [15])

The peak modal gain-current density characteristic is obtained from the following logarithmic relation:

$$g_m = \Gamma g_0 \ln \left(\frac{J}{J_{tr}} \right) \quad (8)$$

as g_0 refers to the optimum gain, J is the injected current density and J_{tr} is the transparency current density. The MQW modal gain is equal to n times modal gain for a SQW, in which n is the quantum wells' number [16,17]. The peak modal gain-carrier density characteristic is given by the following logarithmic relation in Eq. (9) where N is the density of injected carriers [6].

$$g_m = \Gamma g_0 \ln \left(\frac{N}{N_{tr}} \right) \quad (9)$$

4. OPTICAL CONFINEMENT FACTOR

In addition to the various electronic confinement and optical confinement properties, a conventional laser diode has an active region which plays a role as dielectric guide to electromagnetic waves that may be amplified by stimulated emission. In fact, the refractive index of a semiconductor increases when the band gap decreases. Therefore, the diode laser active region refractive index is greater than the limiting regions. With the usual values of the active region thickness and the jump in the refractive index, thus the guide formed is single mode and electromagnetic radiation is closely confined in the active region [18]. Transverse electric mode's optical confinement factor in double hetero-structure is given by:

$$\Gamma_T = \frac{\int_{-d/2}^{d/2} \Phi^2(y) dy}{\int_{-\infty}^{\infty} \Phi^2(y) dy} \quad (10)$$

$\Phi(y)$ is the guided mode profile. It is on a high importance to numerically deal with an eigenvalue equation in the waveguide. For fundamental transverse mode, a remarkable simple expression is as follows:

$$\Gamma_T = \frac{D^2}{2+D^2} \quad (11)$$

D refers to the thickness of normalized waveguide, which is defined by:

$$D = k_0 (\mu_2^2 - \mu_1^2)^{1/2} d \quad (12)$$

where k_0 refers to the vacuum wave number, $k_0 = 2\pi / \lambda$, μ_1 and μ_2 refers to the cladding layers and the active layers refractive indices respectively, and d is the thickness of active layer. For the SQW, the $D \leq 1$ is due to the small active layer thickness. Using Eq. (11) and (12), the confinement factor is simplified to:

$$\Gamma_T = \frac{2\pi^2(\mu_a^2 - \mu_c^2)d^2}{\lambda_0^2} \quad (13)$$

where μ_a and μ_c refers to the refractive indices of the active and cladding layers respectively. The confinement factor for MQW can be calculated by the same way as the DH due to the large number of layers involving tedious procedures. After all, when only fundamental mode is taken in consideration, the MQW structure confinement factor is analogous to a three-layers slab waveguide. However, in the MQW structure, the overall thickness and average refractive index are used. Consider that N_a and N_b refers to the number of active (well) and barrier layers in MQW structure, d_a and d_b (μ_a and μ_b) refers to the thickness of active (well) and barrier layers (and the refractive indices) respectively, $\bar{\mu}$ is the MQW active region average refractive index. Hence the overall thickness and the average refractive index are as in Eq. (14) and Eq. (15) respectively.

$$\bar{d} = N_a d_a + N_b d_b \quad (14)$$

$$\bar{\mu} = \frac{N_a d_a \mu_a + N_b d_b \mu_b}{\bar{d}} \quad (15)$$

The average total effective waveguide thickness is:

$$\bar{D} = k_0 (\bar{\mu}^2 - \mu_c^2)^{1/2} \bar{d} \quad (16)$$

Three-layer equivalent confinement factor is:

$$\gamma = \frac{\bar{D}^2}{2+\bar{D}^2} \quad (17)$$

Therefore, the fundamental transverse mode confinement factor in MQW structure is as follows:

$$\Gamma_T = \gamma \frac{N_a d_a}{\bar{d}} \quad (18)$$

Due to thin active layer (well), the SQW confinement factor is so small in order to improve the optical confinement and decreased the threshold current density MQW structure which is often used in laser diode structure [19].

5. RESULTS AND DISCUSSION

The optical gain is the fundamental parameter characterizing laser diodes when taken into consideration the emission or the absorption of photons in the active region. This factor must

not only be positive for the medium is amplifier, but it has to reach value for cavity losses compensation, so that the laser emission appears. Figure 2 shows the main distinction between the modes of transverse electric and transverse magnetic in GaAs/Al_{0.32}Ga_{0.68}As system.

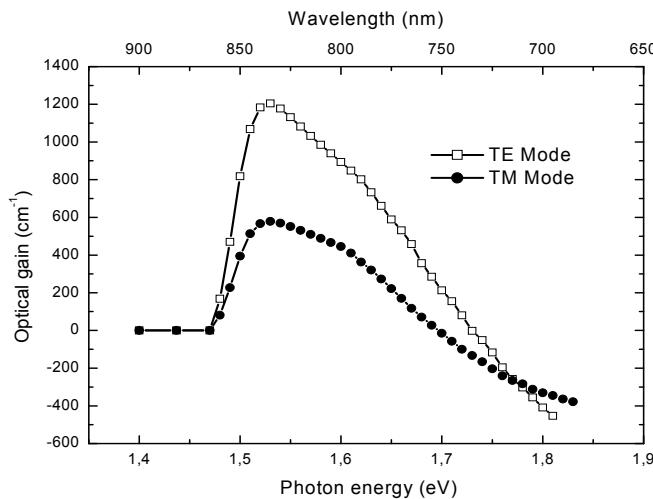


Figure 2. Optical gain spectra in TE mode (white squares) and TM mode (black circles) as a function of: (a) energy photon and (b) wavelength at a fixed injected carrier density of $1.2 \times 10^{18} \text{ cm}^{-3}$, with well of 60 Å thickness and at a 300 K temperature

In Figure 2, both of to heavy hole and to light hole electrons transition are able to be coupled with a polarized optical wave of transverse electric; whose electric vector lies in the plane of quantum well layers. Regardless, the polarized optical wave of transverse magnetic whose electric vector is perpendicular to the plane quantum well layers which permits coupling with the latter transition only. Nevertheless, as the energy level ($n=1$) of heavy hole is of higher density of states comparing to the $n=1$ energy level of light hole, transverse electric wave optical gain is much larger than the transverse magnetic wave optical gain [20]. The peak gain was observed at 1204.2 cm^{-1} in TE mode and 579.1 cm^{-1} in TM mode at the same lasing wavelengths of $\sim 0.811 \mu\text{m}$ and photonic energies $\sim 1.53 \text{ eV}$ respectively at 300 K.

The TE mode (polarized) material gain in the structure in this work at 300 K for various carrier densities is presented in Figure 3.

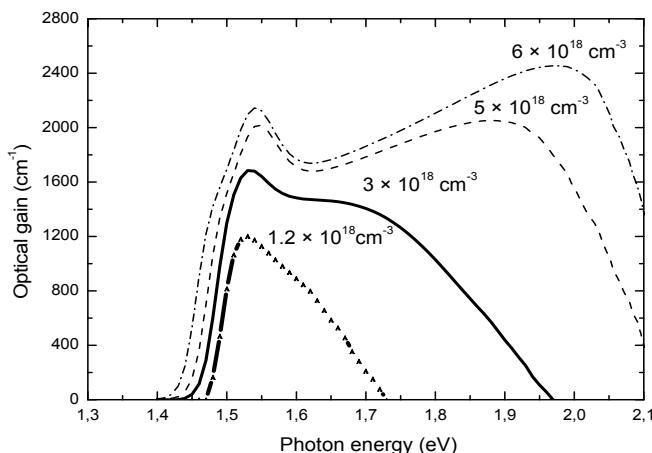


Figure 3. TE optical gain spectra as a function of photon energy for various carrier densities , The injected carrier densities are $1.2 \times 10^{18} \text{ cm}^{-3}$, $3 \times 10^{18} \text{ cm}^{-3}$, $5 \times 10^{18} \text{ cm}^{-3}$ and $6 \times 10^{18} \text{ cm}^{-3}$

In a quantum well structure, the two dimensional nature of the density of states changes the gain curve. In three dimensions, the density of states increases with energy. As a result of injection increases, the gain spectrum peak increases and shifts to higher photon energies with increasing quasi-Fermi level. In contrast, the states' density in a quantum well is constant in the whole subband. Therefore, as the quasi-Fermi level increases under the effect of injection, the second subband is filled at a defined carrier density level. The gain on transition from second subband tends to exceed the gain available on a transition involving the first subband. This can be explained by the fact that the obtained gain is inadequate to compensate losses, the oscillation threshold has not been reached and the second electrons and holes subband must take part in the mechanism. As a conclusion, more levels can contribute to optical gain because when carrier density in active region increases, the quasi-Fermi levels separation increases as well, causing a considerable increase. It can be seen that for low injection level only one peak is obtained around 1.53 eV because only the first subband is occupied at this carrier density injection level and other subband populations are indeed small. As the injection level is increased, two peaks at around 1.55 and 1.89 eV are respectively observed. This could be explained by the simultaneous transition between electrons and both light and heavy holes at this injection level. The gain coefficient progressively becomes larger as a result of the increase in the injected carriers density.

Figure 4 shows the optical gain as a function of photon energy for a SQW and MQW with five wells. The remark drawn from this figure is that the gain of multiple quantum well structure is more important than that of a single quantum well structure. As for the lasing wavelengths, it has a maximum gain at 6020.8 cm^{-1} for multiple quantum well structure and 1204.2 cm^{-1} for structure of a single quantum well. Moreover, the maximum gain tends to shift towards lower wavelengths. This shift is explained by the increase of carrier density which shifts the quasi-Fermi levels and thus increases the energy of transition between quasi-Fermi levels, implying a decrease in wavelength. This analysis concluded that MQW structure provides best gain results for the chosen operating wavelength. By using multiple quantum wells as active region will provoke the optical gain expansion as the maximum gain get on higher standard according to the quantum well structures wells number.

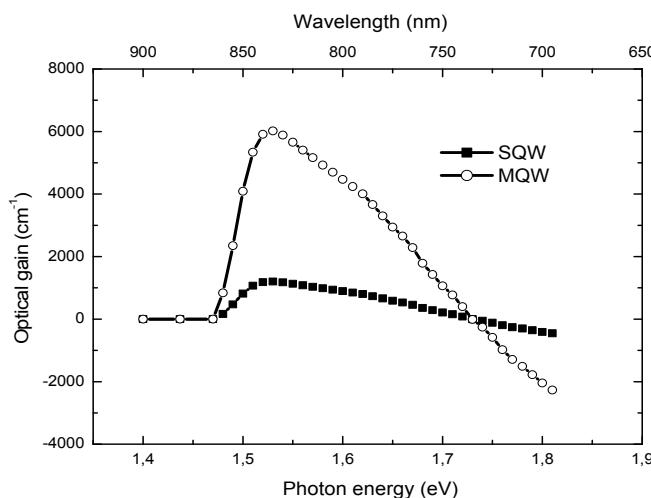


Figure 4. TE optical gain spectra of the two structures: SQW (black squares) and MQW (white circles) at a fixed injected carrier density of $1.2 \times 10^{18} \text{ cm}^{-3}$, with well of 60 Å thickness and at a 300 K temperature.

For further analysis of the two structures, the optical confinement factor was calculated. The well and barrier have refractive indexes of 3.6 and 3.4 respectively. Figure 5 displays the confinement factor variation versus the active layer thickness for various well numbers.

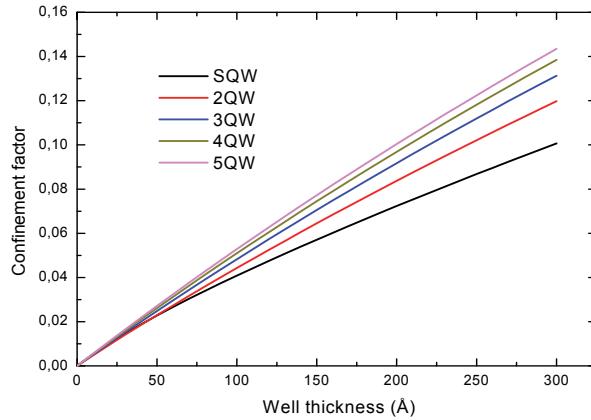


Figure 5. Confinement factor as a function of the quantum well thickness for various quantum well numbers

In Figure 5, it is noted that the confinement factor increment is mainly related to the wells' number and thickness. The calculation is made for only five wells and the maximum value of the corresponding confinement factor is 0.032 for a thickness of 60 Å and it is 0.027 for an SQW laser for the same thickness. So, it is beneficial to use a multi-quantum structure in order to increase the confinement factor. Such increase will give an evolution of the modal gain. It is concluded that selecting the active layer quantum well suitable dimension and wells' number can optimize the structure. Because of size, a poor optical confinement can emerge within single quantum well laser whereas confinement increase considerably for a large number of well. To obtain large confinement factor, the numbers of well needs to be large and the gain should be increased in order to overcome the losses which took place.

In the following section, the peak modal gain is investigated. Γ , in our case is the calculated factor is the same for both transverse electric and transverse magnetic modes. The results of the calculation are shown in Figure 6 which presents the peak modal gain variation versus the injected current and injected carrier density in each of SQW and MQW structures for TE and TM polarization. In Figure 6 (a), the variation is very important for determining the transparency current density J_{tr} , i.e., the value of the injected current density at which the gain becomes positive, and therefore the estimate of the current density threshold. This latter is the value that the current must have so that the gain is equal to the total optical losses of the structure. We note that the gain becomes positive only from a certain value of the current density which is called transparency current density. It is also noted that this value is slightly lower than 48 A/cm². For a significant gain, high current density is necessary and it saturates due to band filling at higher carrier levels. On Figure 6(b), it is obviously clear that modal gain came to be positive only from a certain injection density value which is the transparency density N_{tr} . This density is low for multiple quantum well structure in comparison to that of the structure of a single quantum well. The material transparency is reached when, at certain wavelengths, the stimulated emission exactly compensates absorption, which implies that the beam is not attenuated or amplified in the material. We can see also that the maximum modal gain increases when injected carrier density is increased. On grounds of the high confinement factor values are that high modal gains ones in these structures, because for the MQW structure the confinement factor value is higher than the SQW structure's. This latter is able to decrease threshold current largely but an insufficient carrier confinement problem is brought

by its small physical dimension which will affect the performance of the device. These results agree with the experimental results of E.A.Avrutin et al [3].

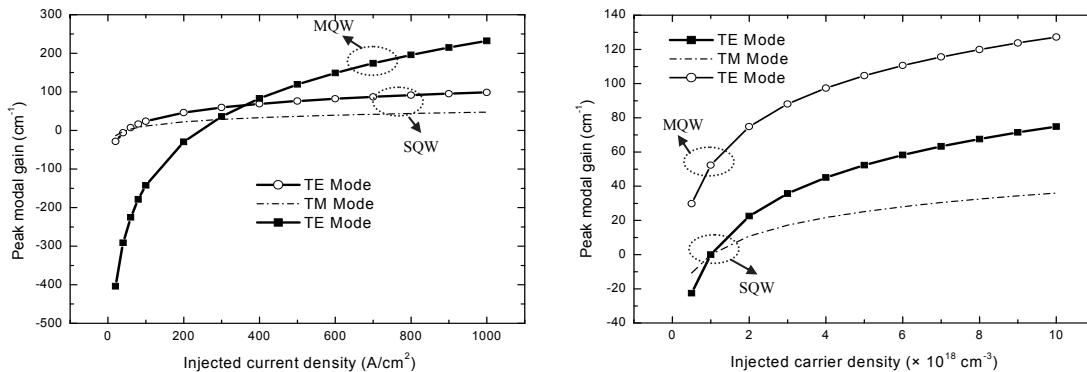


Figure 6. Peak modal gain as a function of: (a) Injected current density for single quantum well (white circles and dashed dot line) in two modes and multiple quantum well structure (black squares) in TE mode; (b) Injected carrier density for a single quantum well (black squares and dashed dot line) in two modes and multiple quantum well structure (white circles) in TE mode

6. CONCLUSION

We have investigated the optical gain and modal gain of a GaAs /Al_{0.32}Ga_{0.68}As QW laser in both polarization modes and the confinement factor has also been investigated. In our calculations, it is demonstrated that the TE polarization and TM polarization gain spectra are not the same. The gain caused by electron-heavy hole transition and electron-light hole transition for the TE polarization is much larger than TM polarization that is caused by electron-light hole transition only. Result also shown that at high injection, within the increase of the separation of quasi-Fermi levels, more levels contribute to the optical gain and it consequently increases. On the other hand, it was found that the multi-quantum-well structure provides very high values of confinement factor compared to those of a structure with a single quantum well. This confinement factor also increases with the thickness of wells; the interest invoked from this increase is to obtain an optical gain and maximum modal gain which are very high. The optical confinement will be poor if the quantum well thickness in a SQW laser is small and such problem can dealt by increasing the wells' number in order to enhance optical confinement factor. So, from the results obtained during in this study, it can be concluded that the performance of multi-quantum well structure is better than single quantum well structure.

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