

## Wake potential of swift ions in copper target

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### Abstract

We investigate the interaction of proton with a solid target, describing the wake effects by taking fitted parameters with experimental values of energy loss function ELF for copper using the dielectric function of random phase approximation (RPA). The results exhibited a damped oscillatory behavior in the longitudinal direction behind the projectile. In addition, the wake potential becomes asymmetric around the z-axis with proton velocity values higher than Fermi velocity  $v > v_f$ , as well as it depends on the position of projectile in cylindrical coordinates .

**Keywords:** Wake potential; Wake phenomena; Dielectric function.

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### 1. Introduction

The subject of interactions of charged particles and matter has received increasing interest in the field of condensed-matter and surface physics over the past years. Especially since the rapid development of semiconductor technology, proton beam writing (PBW)[1], irradiation of quantum dots and welding of nanotubes by ion beams. Material modification, induced by swift heavy ions, has been intensively studied in all kinds of materials, from insulators to superconductors and from crystalline to amorphous materials [2, 3]. The problem of charged particles interacting with solids has received much attention during the past years mainly related to the wake potential, the energy loss and the ion-induced electron emission. By using the dielectric response theory, several authors have studied wake potential for an ion moving on the solid surface [4].

The theory of wake phenomena which concerns the electron density fluctuations excited by charged particles moving in an electron gas has stimulated many experiments [5], In 1940 Fermi pointed out that a swift charged particle traversing condensed matter carries with it an induced electromagnetic field which gives rise to a significant reduction in the energy loss (Fermi density effect). Early studies of this ‘wake’ field focused on particles moving at relativistic velocities [6, 7]. Theoretical estimates were based on the Lindhard ideas [8], and the stopping medium that was considered mostly is the Fermi gas [9-11]. N. Bohr (1948) treated the penetration of atomic particles through matter as shown in Fig.1 where a positive charged particle ( $Z_1e$ ) penetrates a material and moves along the path  $c$ . The induced electric field slows down the projectile, such a field is the result of the induced

polarization of the material. The atomic electrons are slightly displaced in the 'wake' of the projectiles. This effect is strongest behind the projectile than in front of it, and consequently the projectile feels a retarding force ( $Z_1 e E_{ind}$ ), Where  $E_{in}$  is the induced electric field. The retarding force is induced since the field is due to the presence of the projectile, and to estimate the strength of the field  $E_{in}$ , We may simply calculate the electric charge accumulated in the "wake" of the particle, represented by the cone  $A$  shown in Fig. 1 containing the atoms for which the collision is practically completed [12-14].

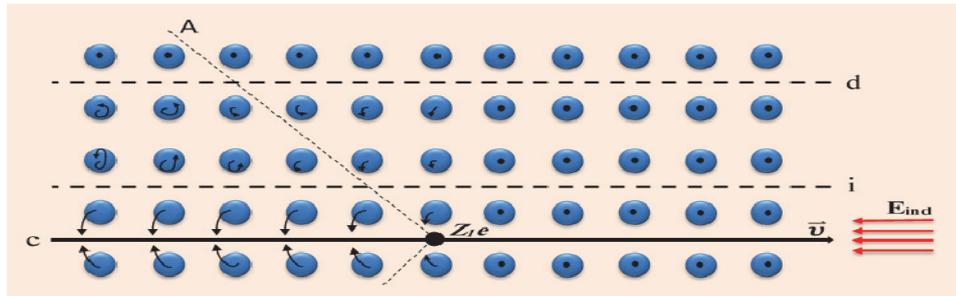


Fig.1: The penetration of charge particle through matter, the polarization in constituent atoms indicated by arrows. The letters **c**, **i** and **d** represent particle

The wake phenomenon received considerable interest in connection with experimental observations on the passage of swift molecules and clusters. A detailed investigation of the wake potential has been made by Isabel Abril *et al.* [8], they found that the position of the dip wake is closer to  $z=0$  when using the more realistic Mermin model compared to the Lindhard model. Subsequent developments made by A. Schinner and P. Sigmund [10, 15] described the wake potential induced by a swift nonrelativistic ion using classical and quantum oscillators.

We have developed a theoretical model and a simulation code to study the wake potential of proton moving in the copper target, in this work, we show the suitability both treatments by comparing their results with available data from experiments. Firstly, we evaluated the wake potential for different values of  $\rho$  coordinates ( $\rho = 0, 3$  and  $5$ ) a.u as a function of distance  $z$  at the certain value of projectile velocity  $v = 5$  a.u. The second procedure to study the relation between wake potential  $\phi_W(\rho, z)$  and  $z$  at different proton velocities spanning values less, around and higher than Fermi velocity  $v_f = 1.12$  a.u., at coordinate  $\rho = 0$ . In the next section, we briefly review the concept of wake phenomena by interaction of swift heavy ion with matter. While the dielectric formalism and the experimental fitted parameters of energy loss function (ELF) to the RPA dielectric model is provided in section three. The result and discussion are presented in section four, atomic units will be adopted throughout the paper.

## 2. Wake potential

The wake potential is a scalar electric potential  $\phi_w(\vec{r})$  in a homogeneous isotropic solid target medium due to swift point charge  $Z_1 e$  moving with constant velocity  $\vec{v}$  [10, 11, 16]. The energy loss of a fast charged particle moving in condensed matter can be described in terms of the complex dielectric function of the target response, which manifests itself as a cylindrically symmetric wake of proton behind the particle, [9, 11]. The wake effect

produced by a proton can be described by the dielectric function  $\epsilon(k, \omega)$  given by the following expression[15]:

$$\phi_w(\vec{r}) = \frac{Z_1}{2\pi^2} \int \frac{d^3k}{k^\epsilon} e^{i\vec{k}\vec{r}} \left[ \frac{1}{\epsilon(k, \omega)} - 1 \right] \tag{1}$$

Where  $d^3k = \left(\frac{k}{v}\right) = dk d\omega$ , therefore equation (1) in the cylindrical coordinates  $(z, \rho)$  becomes:

$$\phi_w(z, \rho) = \frac{2z_1e}{\pi v} \int \frac{dk}{k} \int \omega d\omega e^{i(\omega z/v)} \left[ \frac{1}{\epsilon(k, \omega)} - 1 \right]$$

(2)

Because of the axial symmetry around the path of the moving charge, the cylindrical coordinate  $\mathbf{z}$  and  $\rho$  are sufficient to represent the parallel and perpendicular projections of the vector  $\vec{r}$  relative to the direction of motion. Then the wake potential may be expressed more explicitly as Abril et al (1998) [9].

$$\begin{aligned} \phi_w(z, \rho) = & 2 Z_1 e \int_{\frac{\omega}{v}}^{\infty} \frac{dk}{k} J_0(\rho \sqrt{k^2 - \omega^2/v^2}) \int_0^{kv} dw \\ & \times \left\{ \cos\left(\frac{\omega z}{v}\right) \text{Re} \left[ \frac{1}{\epsilon(k, \omega)} - 1 \right] - \sin\left(\frac{\omega z}{v}\right) \text{Im} \left[ \frac{1}{\epsilon(k, \omega)} - 1 \right] \right\} \end{aligned} \tag{3}$$

Where  $J_0(x)$  is the Bessel function of zero order?

### 3. Dielectric Formalism

The dielectric function describes the interaction of fast ions with the target electrons and nuclei in atoms of matter. The uses of the dielectric formalism to study the energy loss function of charged particles, which reduces gradually its energy, and affects its direction of motion as well as its charge state. The Random Phase Approximation originally derived by Lindhard, was used to treat many properties of the electron gas. Moreover, the general RPA method has had wide applications to the treatment of plasma wake oscillations in finite geometries and in special problems in which other approximations are not applicable at high velocities  $v > v_f$ . The RPA - dielectric function is given by the following equation [5, 17],

$$\epsilon_{RPA}(\omega) = 1 - \frac{\omega_{2p}}{\omega(\omega + i\gamma)}, \gamma > 0 \tag{4}$$

Where the plasma frequency  $\omega_p$  is given by,

$$\omega_p = \left( \frac{4\pi\theta^2 n_0}{m} \right)^{1/2} \quad (5)$$

$\omega$  : is the angular frequency ,  $\gamma$  : the damping constant and  $n_0$  : the electron density.

The set of optical data ( $\omega_i$ ,  $\gamma_i$  and  $A_i$ ) used in this work was taken from reference [18] as shown in **Table 1**, The real and imaginary part(ELF) of the equation (3) are given by following equations[9],

$$\text{Im} \left[ \frac{1}{\varepsilon(\omega)} - 1 \right] = \sum_i A_i \text{Im} \left[ \frac{-1}{\varepsilon_{RPA}(\omega_i, \gamma_i, k, \omega)} \right] \quad (6)$$

$$\text{Re} \left[ \frac{1}{\varepsilon(\omega)} - 1 \right] = \sum_i A_i \text{Re} \left[ \frac{1}{\varepsilon_{RPA}(\omega_i, \gamma_i, k, \omega)} - 1 \right] \quad (7)$$

Table 1: Values of the experimental fitting parameters used the evaluation of equations (6, 7) for the element solid Cu studied in this work [8, 17]

Element	$i$	$\omega_i$ (a.u.)	$\gamma_i$ (a.u.)	$A_i$
Cu	1	0.15	0.04	0.02
	2	0.37	0.22	0.2184
	3	0.7	0.3	0.2449
	4	1.05	0.3	0.1524
	5	2.9	5.6	0.3564

#### 4. Results and Discussion

The Wake potential calculation results for a particle moving in copper for velocity value  $v = 5$  a.u. at  $\rho = 0, 3$  and  $5$  a.u. Using RPA dielectric function, are shown in Figure 2 using cylindrical coordinates. We show the general shape of the wake potential derived from equation (3) which exhibits a damped oscillatory behavior in the longitudinal direction behind the projectile; the pattern of these oscillations decreases exponentially in the transverse direction. In addition, this wake potential extends slightly ahead of the projectile. At coordinate value ( $\rho = 0$ ) the wake potential  $\Phi_w(\rho = 0, z = 0)$  has the lowest negative value (maximum dip depth), because  $J_0(0) = 1$ , We also note that the depth  $\Phi_w(\rho, z)$

increases as the value of the coordinate  $\rho$  decreases, because the wake potential is strongly dependent on  $J_0(\rho, z)$  .i.e, at  $\rho \rightarrow 0, J_0(0) \rightarrow 1$  .

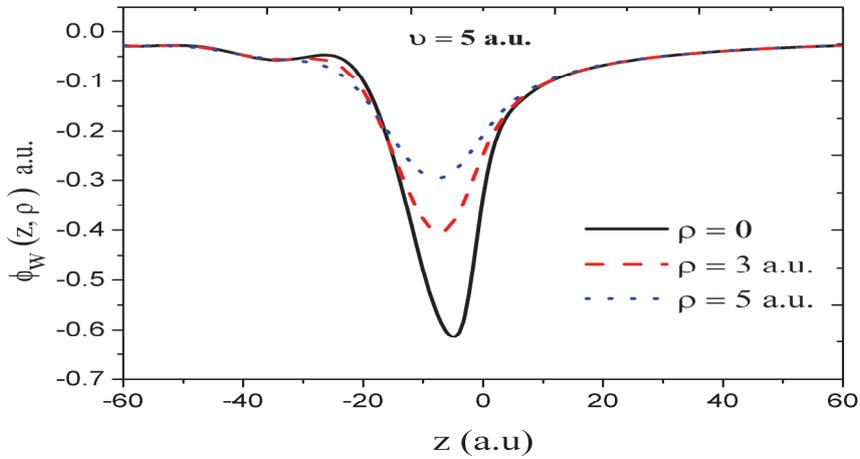


Fig. 2: Wake potential for particle moving in copper with velocity  $v = 5$  a.u. at  $\rho = 0, 3$  and  $5$  a.u., using RPA dielectric function

Fig. 3 shows the results of the relation between  $\phi_W(z, \rho = 0)$  and  $z$  for different Proton velocities spanning values less, around and higher than Fermi velocity  $v_f = 1.12$  a.u. We observe an oscillation in the wake potential in the region behind the proton for velocity values higher than Fermi velocity  $v = 1.5, 5$  and  $10$  a.u. ( $v > v_f$ ). In addition, the depth of the wake potential is observed to be inversely proportional to the proton velocity  $v$  as depicted in Figure 3. The Wake potential depends on proton position ( $z$ ), as well as the proton velocity  $v$ . In the same figure, as the value of the velocity  $v$  increases,  $\phi_W(\rho, z)$  becomes increasingly asymmetric about the  $z$ -axis, also the oscillation in  $\phi_W(\rho, z)$  increases and its depth decreases as the velocity  $v$  increases.

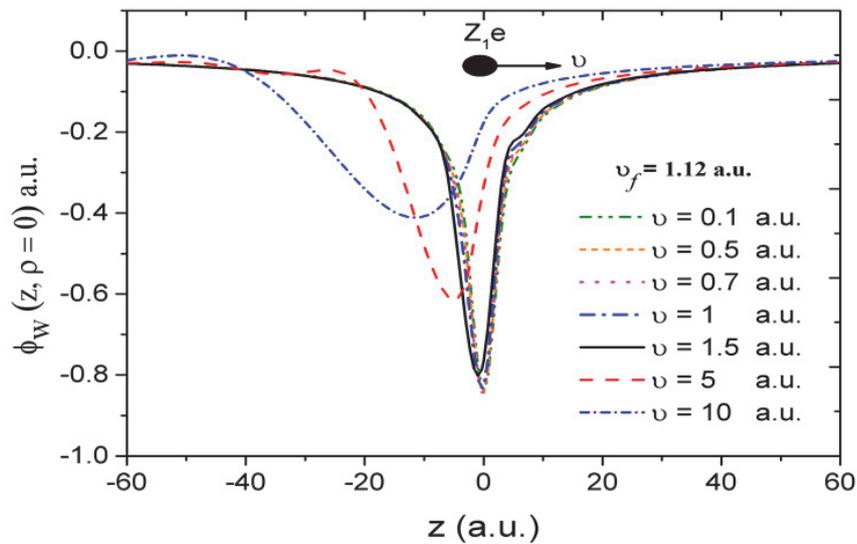


Fig. 3: Wake potential at  $\rho = 0$  plotted as a function of distance  $z$  a.u. for the different velocities higher and lower than Fermi velocity, created by proton moves in copper, using RPA dielectric function

For comparison of present work with others Figure 4 , shows the wake potential for copper target at projectile velocity 1 a.u. , calculated in the present work (solid curve ) compared with Ref [9] (dashed curve). One can notice that there is good agreement between them, both show the same qualitative behavior, the relatively quantitative difference could be attributed to use of different models ,i.e. RPA in our work and Mermin–type ELF in Ref[9],in addition possibly to different computational methods.

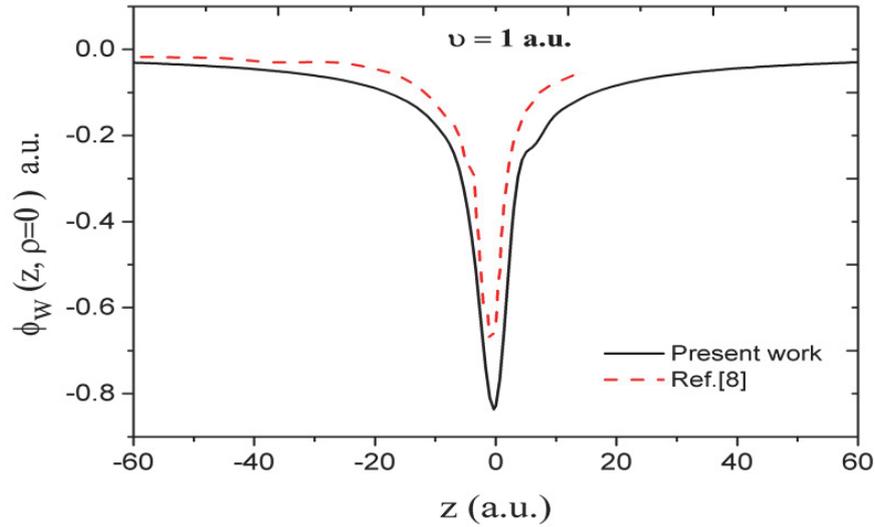


Fig. 4: Wake potential at  $\rho = 0$  by proton moves in copper for velocity 1 a.u. , the solid lines are derived from the RPA- ELF (present work), and the dashed lines are the results obtained with the Mermin –type ELF Ref [8]

## 5. Conclusions

We have presented a theoretical treatment and simulation code to evaluate the wake potential of swift heavy ion moving in the solid target material.

The interaction of proton channeling inside a copper target with the different projectile velocities for the RPA dielectric function has been investigated numerically based on the dielectric response theory of energy loss function ELF. We observed from our calculations, that the oscillating wake effects occur behind the particle opposite to the direction of the particle motion in the solid material and depends on cylindrical coordinate's value ( $\rho, z$ ). The wake potential  $J \Phi_W(z, \rho = 0)$  for projectile velocity less than Fermi velocity  $v < v_f$  (i.e,  $v_f = 1.12$  a.u.), exhibited no oscillations, while for particle velocities higher than Fermi velocity  $v > v_f$  the oscillating wake effects appear behind the particle (i.e,  $z \leq 0$ ). Obviously the asymmetry in the wake potential increased as the velocity increases, and the oscillations of the wake potential developed in the region behind the projectile. Moreover for the wake potential  $\Phi_W(z, \rho = 0)$  with projectile velocity = 1 a.u. , we find a good agreement with previous work based on Mermin type ELF [8]. From comparison of wake potential in the RPA model (present work) with the Mermin dielectric function model, we find that our calculation results in this work are reasonably reliable. In future work, we would like to extend the present work to investigate the interaction of the charged particles with metal films.

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