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Effective Index Method Applied on a GaN/$In_xGa_{1-X}N$ Integrated Optics Waveguide

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ABSTRACT

Effective index is a refractive index which depends not only on the wavelength but also on the mode in which the light propagates. It depends on the structural shape of the waveguide, mainly on its physical dimensions which restrain the light propagation introducing the optical confinement. For this reason a specific method have been developed to evaluate the effective refractive index of waveguides made by a particular wide gap III-Nitrides $In_xGa_{1-x}N$ semiconductor in order to propose a convenient light propagation device since their laser diodes are a reality nowadays and constitutes a first step to develop future all-optical devices. Independently of the wavelength, the smaller is the guiding structure the earlier fundamental and then secondary modes appear.

Keywords: Refractive Index, Effective Indices, GaN/$InxGa1_{-X}N$, Waveguide, Integrated Optics.

1. INTRODUCTION

Refractive index can be considered as a speed ratio of an electromagnetic wave related to its speed in vacuum. It is an intrinsic property of a given material. Waves travel slower through physical materials with higher than vacuum refractive index [1].

Now, an effective refractive index appears in a light guiding medium as a measure of the overall delay of a light beam in that component. It is no more a proper property of the optical material is made of but an effective property of the optical material altogether with given physical dimensions. Obviously, the effective index is not just a material property, but depends on the whole waveguide design.

Then, the effective index states the ratio of the velocity of light in vacuum to the velocity of a given mode for a given polarization in the direction of propagation in the guiding structure along the waveguide in the z direction.

The effective index method is a very efficient tool designed in order to transform a two dimensional electromagnetic field problem to a one dimensional effective waveguide. This method was used as early as the late 70's applied to semiconductor lasers having a gradual lateral variation in the complex permittivity [2].

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Others developed and used other methods to determine this very important characteristic. Yet, they developed a method for 3D-to-2D dimensionality reduction of scattering problems in photonics. They found that contrary to the standard effective index method the effective parameters of the reduced problem were always rigorously defined using the variation technique based on the vectorial 3D Maxwell equations. Applied on a photonic crystal slab waveguide, they showed that this approach predicted well the location of the bandgap and other spectral features much more precisely than any standard EIM approximation [3, 4].

Precise modeling of optical structures requires 3D calculations are very voracious in terms of time computation and particularly memory requirements. Bostan et al. showed that effective index approximation of Photonics Crystal Slabs waveguides has been used for relieving these requirements by reducing the full 3D calculations to simpler, though approximate 2D calculations [5]. The effective refractive index n_e of a heterogeneous material characterize the phase velocity which is a property of the coherent or forward-scattered wave. It is of an extreme importance for a correct interpretation of optical experiments and some researchers demonstrated that it can be obtained in strongly scattering samples by measuring the angular-resolved transmission [6].

Moreover, an asymptotic analysis of the effective-index method applied to linear and plane arrays of identical rectangular waveguides by two different ways of applying the method, depending on how the effective index is calculated were also considered. The expressions were also used to highlight the effects of the dimensions of the array on the accuracy of the methods [7]. The same author established an effective-index method with built-in perturbation correction for the analysis of the vector modes of general rectangular-core optical waveguides. This method preserved the simplicity and the high efficiency of the conventional EIM but produced significantly more accurate results [8]. These latter developed also an effective-index method (EIM) with built-in perturbation correction for the analysis of the vector modes of general rectangular-core optical waveguides. They assumed that this method maintained the simplicity and the high efficiency of the conventional EIM, yet producing significantly accurate results [8].

Earlier on a four-layer waveguide a method comprising the complex effective index was developed for calculating the threshold gain in distributed feedback [9]. Further, Xie et al. used the Effective Index Method (EIM) in order to model complex fibers geometries like the multifilament core fibers considered as step-index fibers and applied it successfully to both index-guiding photonic crystal fibers and all-solid photonic bandgap fibers [10].

Therefore some reported that in standard EIM, a relatively large error in predicting the resonant mode locations was often seen as a result of large offset in simulated photonic bandgap. This lead to believe that accurate prediction of resonant peak locations from 2-D FDTD was feasible by properly adjusting the effective index to matching the simulation photonic bandgaps [11].

A novel effective index optical model was used for the analysis of lateral waveguiding effects in vertical-cavity surface-emitting lasers. Not only reducing the dimensionality of the Maxwell equations describing the lasing mode, this model also granted new insights into waveguiding phenomena in vertical-cavity lasers. Particularly, the effective index responsible for waveguiding was dependent only on lateral changes in the Fabry–Perot resonance frequency [12].

An improved effective index method was also employed with the objective of evaluating its limitations for various designs of long period gratings photonic crystal fibers compared with the corresponding values of multiple multipole method. They showed that this method seems to be excellent when the surrounding media was assumed to be air, when it becomes less accurate when the fiber was immersed into a liquid with a refractive index close to that of the cladding [13].

Maybe cause of the lack of data concerning the optical guiding properties of structures made on III-V semi-conductor alloys or by the fact that actual optical guides give sufficient performances, one did not found any related paper concerning the usage of these devices as photonic crystal waveguides.

2. MATERIALS AND METHODS

The considered waveguide is schematized in the figure 1. It consists on a guiding structure of "n_1" refractive index buried in a cladding structure or substrate of "n_2" refractive index less than "n_1" for optical confinement necessity.

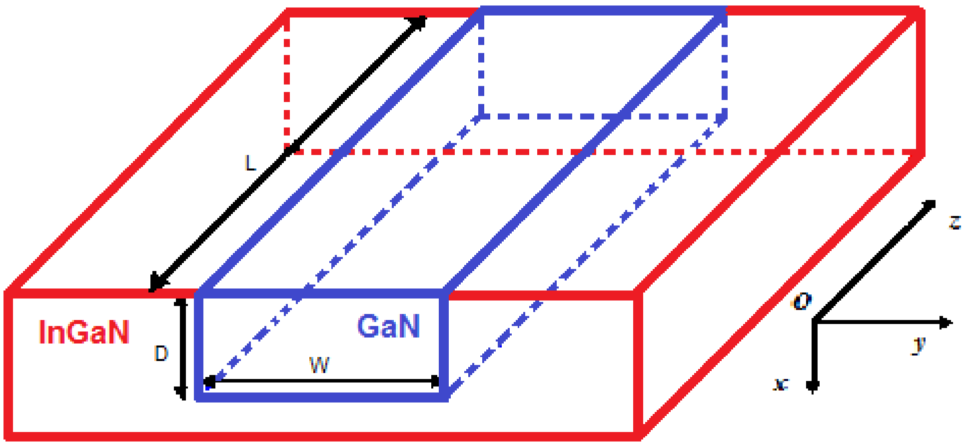

Figure 1. Structure of the considered waveguide.

The guiding structure, where the light will remain confined is the GaN core. Light will propagate along the structure in the z-direction remaining inside the GaN structure. There are, all along, optical reflections in the limits of GaN and InGaN in the substrate but the light no more could get out the guide since the refractive index of the core is greater than the refractive index of the air. It occurs several "modes" of propagation due to the inclinations of the incident light. These modes propagate with different speeds materialized with different refractive index. These refractive index are then not only depending on the way light is introducing in the guiding structure but also on the dimensions of this latter. One obtains then particular "effective" indices in both quasi Transverse Electric modes and quasi Transverse Magnetic modes.

In the first approach, the step index waveguide schematised in the figure -2- is supposed to have the dimension "w" tending to the infinite [13].

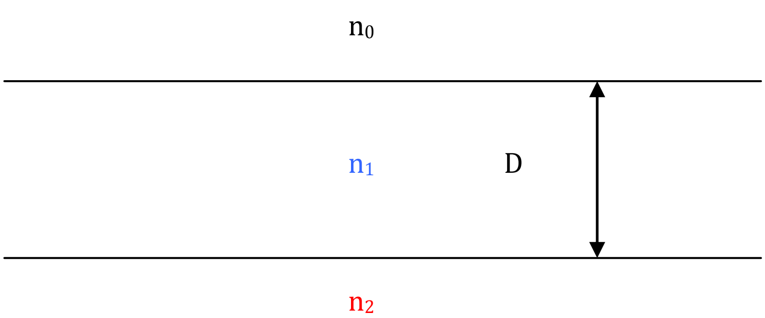

Figure 2. Structure of the planar waveguide.

The calculation of the effective index of the planar guide obtained for each propagated mode is easy. In a second step, the obtained index is taken as the vertical guide index, taking the dimension "D" tending to the infinite, now.

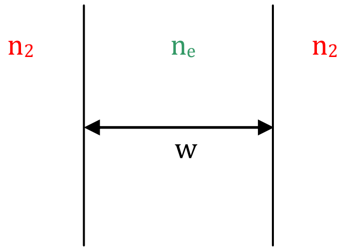

Figure 3. Structure of the vertical waveguide.

The effective index obtained is thus the effective index of the original waveguide. This method can also be applied to the graded index waveguides, because the gradient can be obtained on both x and y transversal variables.

The Effective Index Method or EIM permits, then, to replace an original 3D linear rectangular waveguide schematized in figure -1- with an equivalent planar guide where the index profile to be determined will only depends on y and z co-ordinates, without significant loss of crucial information. This index becomes efficient and is called effective index since it depends on the physical and optical dimensions of the waveguide no more on the material involved only.

Therefore some conditions remains to use this way of resolving the problem. The main one is that the effective index method could be applied only when the electromagnetic field could be expressed with a variable separation so that the electrical field could be decomposed with a separation of the transversal variables as [14-17]:

$$\emptyset(x, y, z) = f(x).g(y).e^{j(\omega.t-\beta.z_0)} \tag{1}$$

This gives in two dimensions:

$$\Phi(y,z_0) = g(y)\ .e^{j(\omega.t\ -\ \beta.z_0)} \tag{2}$$

Then, both Fast Direct and Inverse Fourier Transform give the electrical field in a so simple manner in $z_0+\Delta z$. Nevertheless, "n_e" the effective index have to be already calculated for every equivalent guided mode in the waveguide. The procedure consists in evaluating the Fast Fourier Transform applied to the function in "y" variable g(y).

There we obtain F(kx) which is multiplied by the "propagation term $e^{j\beta\Delta z}$", where $\beta=2\pi/\lambda_0$ is the propagation constant of the propagated mode, "n_e" the calculated effective index, λ_0 the beam wavelength and "Δz" the z-axis progression step. Finally one calculate the Inverse Fast Fourier Transform of each sample to obtain the electrical field at $z_0+\Delta z$.

$$\Phi(y,z_0+\Delta z) = f(x)\ .g_{\Delta z}(y)\ .\ e^{j(\omega.t\ -\ \beta.(z0+\Delta z))} \tag{3}$$

$g_{\Delta z}(y)$ is the new electrical field repartition related to the x and the $z_0+\Delta z$ co-ordinates.

Concerning the effective index calculation the method consists to resolve the dispersion relation for the step index waveguides, given by:

For TE modes:

$$k_0.h.(n_1^2 - n_2^2)^{\frac{1}{2}} = arctg\left[\frac{n_e^2 - n_0^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}} + arctg\left[\frac{n_e^2 - n_2^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}} + m.\pi. \quad (4)$$

For TM modes:

$$k_0.h.(n_1^2 - n_2^2)^{\frac{1}{2}} = arctg\,\frac{n_1^2}{n_0^2}\left[\frac{n_e^2 - n_0^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}} + arctg\,\frac{n_1^2}{n_0^2}\left[\frac{n_e^2 - n_2^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}} + m.\pi. \quad (5)$$

"m" is the mode number, $k_0 = 2.\pi / \lambda_0$ the propagation constant, λ_0 the wavelength and "h" is the film thickness. This is an equation of two unknown "n_e" and "m". The wave guiding condition remains: $n_2 < n_e < n_1$, remembering that n_2 is the substrate index where n_1 is the film index. There are, thus, total reflections of the electromagnetic wave on the guide bonds. The light remains confined inside the guide.

To solve the problem and obtain the effective refractive index one has to use the so called Bolzano's dichotomy method, consisting in the searching of the zeros of a function.

The convenient procedure is that if one have a function as $f(n_e) = 0$, and if we choose two arbitrary values n_{e1} and n_{e2}, such that:

1) $f(n_e)$ is monotonous in $[n_{e1},n_{e2}]$ range means that $f(n_e)$ is increasing, decreasing or at the extreme equal to zero in this interval.
2) $f(n_{e1}).f(n_{e2}) < 0$, implies that the derived of $f(n_e)$ changes of sign in the above mentioned interval.

Then, the suite:

$$n_{e_{n+2}} = \frac{n_{e_{n+1}}.f(n_{e_n}) - n_{e_n}.f(n_{e_{n+1}})}{f(n_{e_n}) - f(n_{e_{n+1}})} \quad (6)$$

Have n_e as limit, such that $f(n_e) = 0$ when n tends to infinite.The maximum number of guided modes in a given structure of "h" thickness is in consequence:

For TE modes:

$$m_M = \frac{k_0.h.(n_1^2 - n_e^2)^{\frac{1}{2}} - arctg\left[\frac{n_e^2 - n_0^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}}}{\pi} \quad (7)$$

For TM modes:

$$m_M = \frac{k_0.h.(n_1^2 - n_e^2)^{\frac{1}{2}} - arctg\,\frac{n_1^2}{n_0^2}\left[\frac{n_e^2 - n_0^2}{n_1^2 - n_e^2}\right]^{\frac{1}{2}}}{\pi} \quad (8)$$

3. RESULTS AND DISCUSSION

A considerable interest in column III elements (like B, Al, Ga and In) combined to form a Nitride semiconductor materials with "N" as the V column element has appeared early and this for their so suitable optoelectronic properties [18]. Their main characteristic is that in the Wurtzite (Hexagonal) configuration these semiconductors have direct bandgaps optimizing their optical properties such as radiations and direct transitions compared to the other indirect bandgaps III-V semiconductors.

The semiconductors compounds in InGaN ternary system are of both fundamental and practical interest in the realisation of devices like p-n junctions, transistors, photo-detectors, injection lasers, quantum wells and LEDs [20,21]. These semiconductors keep direct bandgaps throughout the alloy composition range, and the direct energy gap of these materials covers a wavelength range from the infrared to the near ultraviolet. They also have very large energies of formation and are thus useful for high power and high temperature semiconductors devices [22].

Strong atomic bonding and very large energies of formation, making them potentially useful to make light-emitting diodes, injection lasers, high power and high temperature devices [23]. Their energies correspond to emission wavelengths of 1.55 μm infrared or the well-known "telecommunication wavelength" by excellence, to 0.197 μm mid-range ultraviolet. This variety of the light spectrum makes them well applicable for short visible wavelengths area to the UV optoelectronic largely greater than the conventional III-V semiconductors covered field [17].

Both –n and –p types layers could be crystallized and the commercial blue diode based on InGaN/GaN was early available [24].

Practically, when a semiconductor is under an electromagnetic ray it absorbs a quantity of optical energy equals to "hν", where "h" is the Planck constant and "ν" the wave frequency. This boosts the electrons of the valence band to jump to the conduction band. Also, once the wave velocity is increased the number of electrons will be greater making the semiconductor more conductive. Then, the width of an energetic authorised band is proportional to the exchange integral which increases with the electrons energies. By this fact, the width of the bandgap is modified, since by interactions the conduction band is attracted to the valence band and the bandgap value decreases. The influence of the composition has shown that it is possible, acting on the dosage, to control the alloys refractive indices by their bandgaps. It is also possible to obtain a precise refractive index in choosing, judiciously, the binary compounds used in the ternary alloy composition [18]. According to these assertions, in the first time one has calculated the effective indices of the infinite planar waveguide considering the superstrate index as air once the GaN waveguide index equals 2.484 at the wavelength of 0.359 μm [19].

Almost whole part of electromagnetic guided wave energy is confined in the fundamental modes called TE_0 and TM_0. Each mode possesses its effective index corresponding to a different speed, lower than the fundamental one. Thus the main goal, designing an optical waveguide, is to obtain only one guided mode, corresponding to the fundamental one.

In the present work instead of varying the waveguide width, one changed the stoechiometric coefficient. Then, one has to continuously evaluate first the infinite planar waveguide and there injects it in the vertical waveguide.

In a previous work, one had evaluated effective indices by modifying the width of the waveguide [17]. Taking a depth of 1 μm or 1000 nm, one have initially calculated the effective index of the planar guide and found an effective index of ne_{TE0}= 2.465 corresponding to the transversal electric quasi mode, and also another effective index ne_{TM0} = 2.473 related to the transversal magnetic quasi mode. Then, one have used these indices as the planar guiding structure indices in both cases and took the superstrate and the substrate indices equal to the substrate one which is in occurrence equals to 2.16 when refractive indices of $In_xGa_{1-x}N$ have already been calculated in a previous work according to a bowing parameter of 2.17 [18].

Nevertheless, to a better integrating rate one have taken a depth of just only 0.1 μm or 100 nm and found a serious difference in effective indices. Using several width of the considered waveguide, both transversal electrical TE and transversal magnetic TM modes have been involved. The reason of the choice of a fixed depth and a varied width is that it is further easier to modify the width of the mask in the growing process than the epitaxial growth thickness. Hence, the depth had been taken as 0.1 μm which can be controlled in the duration of the growth process; varying the size of the mask applied on the structure one can obtain several widths. Remark that the structure has to be strictly linear.

Theoretically, calculations began by the apparition index width corresponding to the first guided mode till a width of 0.45 μm, and this for both TE and TM modes, all along. It is finally obtained a whole range of the waveguide's effective indices. Results are confined in the figure 4. Concerning TE modes, the first or fundamental mode appears at a width of just only 0.005 μm so 5nm, when for TM modes the fundamental mode appears at 0.016 μm so 16 nm. These widths are called the cut off or extinction widths. Below these widths, light beam diffuses in the substrate and this mode is not a guided one. Augmenting progressively the waveguide width one obtained the other modes called secondary modes. The following modes begin as far as 25 nm for TE modes and at 47nm for TM modes.

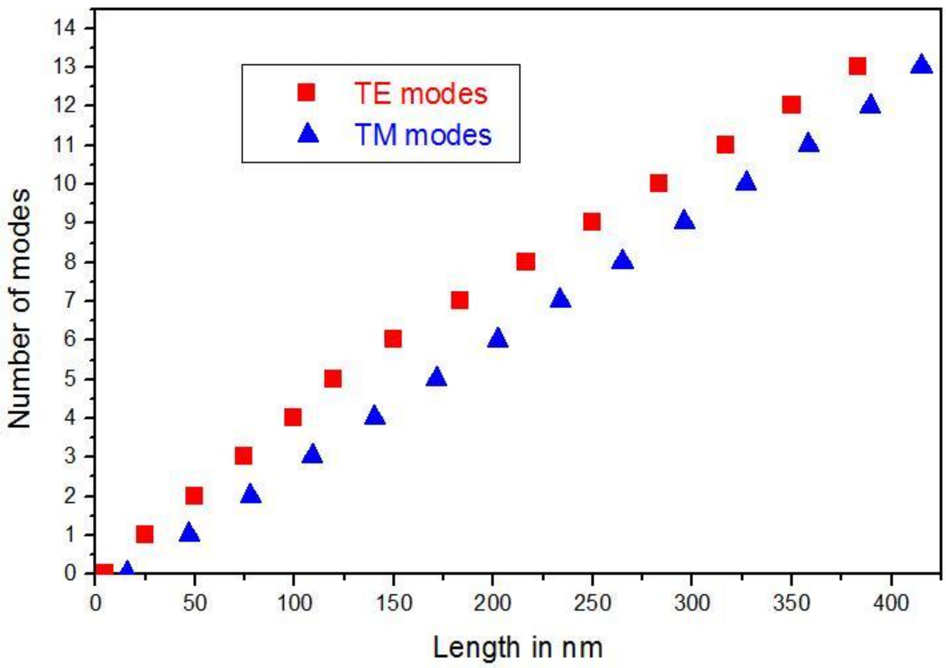

Figure 4. TE and TM modes.

Continuing to enlarge the waveguide it could be found that effective indices which increase gradually tend to a maximal value never reaching it materialized by the "n_1" guiding structure original index. There is then, an asymptote which cannot be trespassed, and this even if the waveguide width reaches an impossible infinite value.

Then, to use a single mode or monomode waveguide one have to take a width less than 25 µm, or the first secondary mode will appear and subtracts energy from the guided mode. If the width is augmented, the amount of guided energy will be subtracted every time another mode appears.

The following table 1 shows the results and show exactly the dimensions of every mode apparition.

Table 1. Widths in nm of apparition of the different TE and TM modes

Mode	0	1	2	3	4	5	6	7	8	9	10	11	12	13
TE[nm]	5	25	50	75	100	120	150	184	217	250	284	317	350	384
TM[nm]	16	47	78	110	140	171	202	234	265	296	327	358	389	418

In conclusion, from an initial extinction value to an asymptotic right the dimension which guide the apparition of guided modes depends mainly on the initial dimensions of the guided structure. Independently of the wavelength, the smaller is the guiding structure the earlier fundamental and then secondary modes appear. The same way of thinking is applicable for TM modes. This means that to design a single-mode or monomode waveguide it would be enough to limit the height of the optical waveguide to 16 nanometers. Most of the electromagnetic wave will propagate in this monomode waveguide and there will be no delay and no phase difference in the optical wave that will arrive at the end of the optical chain.

4. CONCLUSION.

This paper has calculated effective indices of a GaN/$In_xGa_{1-x}N$ structure, even if this semiconductor seems to be very absorbent. This means that to design a single-mode or monomode waveguide it would be enough to limit the height of the guide to 16 nanometers. Most of the electromagnetic wave will propagate in this guide and there will be no delay no phase difference in the optical wave that will arrive at the end of the optical chain. Then, replacing gold connections in a microchip could not only reduce the electrical power consumption but also reduce greatly the costs of such devices, not talking about the speed of communication which will be increased tenfold.

Reducing the physical dimensions of the waveguide, one had obtained a guided mode from the very small value of 5 nm which seems to be very interesting in a practical sense. This showed that even with a wavelength of 0.359 µm a guided mode could appear at a very small dimension leading to a better integration of future integrated optics.

This kind of optical waveguides could be used in an all-optical device where the emitter transmits directly incident light towards the receptor instead of using electrical connections.

Nevertheless, because of the great absorbance of such a structure, one has to use this kind of semiconductor just at very short distances. Therefore, the obtained results showed that it is suitable for short communications in a single chip of an integrated circuit and using GaN/InGaN laser emitter and photodiode bond with this kind of waveguide could not only save a great amount of electrical energy but reduce the cost of the microchip and accelerate the transfer speed of information.

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